Housing Price and Fundamentals in a Transition Economy: The Case of the Beijing Market*

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Abstract

This paper develops a dynamic rational expectations general equilibrium framework that links house value to fundamental economic variables such as income growth, demographics, migration and land supply. Our framework is general and can handle non-stationary dynamics as well as structural changes in fundamentals that are commonplace in transition economies. Applying the framework to Beijing, we find that the equilibrium house price and rent under reasonable parametrizations of the model are substantially lower than the data. We explore potential explanations for the discrepancies between the model and the data.

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1 Introduction

Economists have long been interested in the relationship between house price and economic fundamentals. With a few exceptions, much of the literature has focused on developed economies.[1] Recently, the unprecedentedly high and rising house price in major Chinese markets has attracted global attention (Fang et al., 2016). For example, between 2006 and 2014, real house price in Beijing appreciated by 19.8% per year (Wu et al., 2016). This is more than double the annual price growth in Las Vegas – itself one of the fastest appreciating markets in the U.S. in recent years. Yet Beijing has also experienced an extraordinary growth in income and population – with an average annual real disposable income growth rate of approximately 9% and a net annual inflow of migrants that is about 4% of the incumbent population during the last decade.[2] While qualitatively these growing fundamentals may support the high level of Beijing house price today, it is challenging to evaluate such justification quantitatively, especially given the non-stationarity in the economic fundamentals often encountered in a transition economy.[3] As noted by Robert Shiller in April 2014, "China is in such a rapid growth period. It is very hard to price assets when growth is at the high level. The future matters more. In a stable economy that is not going anywhere, you have a pretty good idea of what they are worth." In a transition economy, the historical relationship between house price and fundamentals is unlikely to repeat itself. Conventional house pricing methods, such as price-income and price-rent ratios, often fail to capture the changing dynamics in the transition phase and hence are insufficient for assessing house price in transition economies.

Instead, in this paper, we develop a dynamic rational expectations general equilibrium model with the goal of understanding the relationship between house price and fundamentals in a transition economy. The model is general and flexible enough to deal with situations where fundamentals can be non-stationary and rapidly changing, yet retains tractability for deriving the price-fundamental relation endogenously and dynamically along the transition path.

The model features overlapping-generations of heterogeneous households on the demand side and a representative housing production firm on the supply side. Housing in our model is not only consumption but also investment. Housing demand is determined both by the

[1] See, e.g., Capozza et al. (2002), Ambrose et al. (2013), Glaeser and Gyourko (2005), Himmelberg et al. (2005), Hott and Monnin (2009), Poterba et al. (1991) and Case and Shiller (2003).

[2] The statistics are computed from the National Bureau of Statistics of China.

[3] Throughout this paper, we refer an economy that is undergoing structural transformations as a transition economy.

extensive margin (renting vs. owning) and the intensive margin (the amount of housing to consume/invest). Leverage is allowed through mortgage for households who have enough savings to meet the down payment. There is no aggregate risk in the model – agents have perfect foresight about the future dynamics of the fundamentals and take them into account when making current consumption, investment and production decisions. Following Aiyagari (1994), the model admits of two types of idiosyncratic risks: household's income shocks and medical expense shocks. Thus heterogeneity among households stems not only from age but also from income and expenses. The idiosyncratic risks are uninsurable and lead to additional precautionary demand for housing.[4]

We apply this model to Beijing. In doing so, we are able to capture a wide range of important and well-documented features of the Beijing housing market: (i) income growth has been high but is expected to slow down (Pritchett and Summers, 2014); (ii) urbanization process is on-going but is expected to stabilize eventually (Garriga et al., 2016); (iii) population is aging in the near term (Song et al., 2015); (iv) access to mortgage loans is allowed with a relatively high down payment requirement (Fang et al., 2016); (v) urban land supply is at the control of the government (Cai et al., 2013); and (vi) investment vehicles are limited (Chen and Wen, 2015). These features interact in a general equilibrium setting, yielding evolving dynamics of the fundamentals over the course of the transition period. Once the economic transition is complete, the economy enters a balanced growth path (BGP) that is assumed to resemble the current state of a reference city such as Hong Kong or Washington, D.C. In the BGP, per capita income, land supply and population grow at constant rates, leading to constant growth in house price, rent and housing supply. Using the equilibrium quantities in BGP as the terminal conditions and backward inductions, we solve for the trajectories of equilibrium house prices and rents during the transition phase.[5]

Our analysis produces consistent and robust findings. First, using Hong Kong as a reference city for the future BGP, the baseline model predicts that the equilibrium Beijing house price in 2014 is about 30% below the observed market price. To probe the robustness of the finding, we experiment with alternative assumptions about land supply, income growth, population structure, mortgage rates, and down payment constraints. We also examine an alternative specification where Washington, D.C. is a reference city. We further extend the

[4]The precaution demand induced by households' income and medical expense shocks is found to be quantitatively important in our calibration exercise, consistent with the existing literature ((Chen, 2010), (Gan, 2010) and (Iacoviello and Pavan, 2013)).

[5]In the application of our model to the Beijing housing market, the transition phase lasts 110 years. We develop an efficient numerical method to compute equilibrium house prices and rents during this period.

model to allow residents in second-tier cities to endogenously choose between staying in home cities and moving to Beijing.[6] In all these cases, we observe moderate changes in the equilibrium Beijing house price in 2014; however, the gap between the predicted and actual house price remains substantial.

Second, despite limited land supply and influx of migrant workers that keep up the housing demand, the Beijing market becomes more affordable over time, as evidenced by the increasing ratio of average annual income to price per square meter. Thus, high price-income ratio *alone* does not indicate that house is overvalued. Instead it may be consistent with the evolution of economic fundamentals and may converge to those found in developed markets as the economy matures. It is therefore misleading to compare price-income and price-rent ratios in a transition economy over time or across countries because these ratios are evidently not stationary during the economic transitions.

We stress that our analysis is meant to develop a dynamic equilibrium framework to examine house price in a non-stationary environment with changing fundamentals rather than to fully account for every single factor in the Beijing market. In this sense, our findings say little about whether there is a housing bubble in Beijing. Aside from the possibility of a bubble, the discrepancy between the model-implied and market-observed prices could be driven by factors that have not been addressed in the current model, or reporting errors of fundamentals, or both. On one hand, we abstract from a number of market factors to focus on the price-fundamental relationship in a non-stationary economy. These factors include constant interventions from the Chinese government (Wu et al., 2012), endogenous rural-to-urban migration (Garriga et al., 2016), aggregate uncertainty (Piazzesi et al., 2007; Han, 2013), illiquidity of housing e.g. (Haurin and Gill, 2002; Han, 2008) and demand shocks and search frictions (Genesove and Han, 2012; Head et al., 2014). Each of these factors could bring additional dynamics into the housing market.[7] On the other hand, a recalibration of our model indicates that, all else equal, to justify the observed high house price level, the actual average income in Beijing would need to be about 60% over the income as officially reported by the National Bureau of Statistics. This is not entirely unrealistic in light of the recent evidence suggesting that official reports by the National Bureau of Statistics may

[6]In doing so, we omit rural-urban migration, which is shown to be a significant factor that accounts for house price growth in Chinese markets in Garriga et al. (2016). We discuss the consequence of such omission in our framework in Section 5.4.2.

[7]To gain some insights, we perform counterfactual exercises to examine some form of the aforementioned factors –such as building restrictions, income growth shocks, and mortgage policies – and find that they do not help close the gap between the equilibrium model price and the market price.

significantly understate income for Chinese government officials (Deng et al., 2016). However, such substantial income understatement cannot represent the overall population, nor can it be sustainable in the long run. While accounting for additional elements – whether other potential factors or measurement issues – in the current framework is beyond the scope of this paper, we believe it would be a promising avenue for future research. In this sense, we view our model as a useful starting point for studying the implications of the evolving fundamentals of housing markets in transition economies.

This paper is organized as follows. Section 2 reviews the related literature and highlights our contributions. Section 3 presents the model and discusses properties of the economy in the BGP. Section 4 specifies the dynamics of exogenous fundamental variables and calibrates the model using the Beijing housing market data. Section 5 reports quantitative results for the model equilibrium outcomes such as price and rent under the baseline scenario, a variety of robustness checks as well as an extended version of the basic model. Section 6 concludes the paper.

2 Literature Review

This paper is most closely related to an important and growing literature that aims to understand house price growth in developing economies with China as a leading example. On the empirical side, Wu et al. (2012) provide an assessment for eight major Chinese housing markets during 2003-2010. They find that high expected house price appreciation is needed to justify the low rental return and that much of the increase in the Beijing house price occurs in its land values; both findings are equilibrium outcomes in our structural model. Using an independent data source, Fang et al. (2016) document the patterns of house price growth and local per capita aggregate income growth for 120 Chinese cities during 2003–2013. Among other things, they find that Beijing house price levels increased 660% accompanied by rapid but declining income growth and changing demographic trends. Their findings provide a natural motivation for our model. Together, by highlighting the unique features of the Chinese housing market, these papers have established inspiring evidence that makes a strong case for understanding the ongoing Chinese housing boom in a dynamic general equilibrium setting.

Theoretical work that models house price path in a transition economy has been relatively limited. Two notable exceptions are Chen and Wen (2015) and Garriga et al. (2016). Chen and Wen (2015) present a theory of rational bubble and show that high house price growth relative to income growth in China indicates a bubble that emerges from resource reallocation from the traditional low-productivity sector to the emerging high-productivity sector. More recently, using a dynamic general equilibrium rational expectations model, Garriga et al. (2016) show that the quick rise of house price in China between 1998 and 2007 can be largely explained by the process of rural migrants entering cities driven by ongoing structural transformations. Both models are attractive because they not only endogenize the massive labor movements across the regions/sectors featured in the economic development stage but also do a good job in quantitatively replicating the house price dynamics.

Built upon this line of literature, our study focuses on the equilibrium relationship between house price and fundamentals in a non-stationary economy. Unlike Chen and Wen (2015) and Garriga et al. (2016), we focus on one city in China – Beijing, and take its population inflow as exogenous in the main analysis.[8] While this prevents us from investigating the labor market dynamics, it helps us examine the role of a rich set of fundamentals, including rapid yet declining growth of income, aging population in the near term, ongoing urbanization, limited land supply, and financial market frictions.[9] These factors are among the most important facing the current Chinese housing markets, as emphasized in Fang et al. (2016) and Wu et al. (2012). In addition, our setting is flexible enough to account for the unique features of housing itself. For example, we model housing demand both at the extensive margin and at the intensive margin. Both consumption and investment roles of housing are important in our model. This is in contrast with Chen and Wen (2015), who treat housing as an investment only and Garriga et al. (2016), who mainly focus on the consumption role of housing.

More broadly, this paper is related to a rich body of literature that studies the structural link between house price and fundamentals in developed economies. Much of the research in this area focuses on the U.S. market. For example, using a dynamic general equilibrium model, Sommer et al. (2013) and Chu (2014) find that a large fraction of the observed house price increase in the decades since 1995 can be explained by the changes in credit constraints, low interest rates and growing income. Introducing search and match friction into the housing market, Head et al. (2014) use a dynamic search model to understand the short-run dynamics of average house prices, home sales, construction and population growth.

[8] We offer an extension that endogenizes the migration flow from second-tier cities to Beijing in Section 5.4.2.

[9] For example, our model takes two dimensions of population structure into account: population size and age distribution. The latter is important because housing demand has a clear life-cycle profile, as shown in Yang (2009).

In addition, using a classical asset pricing model, a number of papers examine how interest rates, dividends or rents affect house prices (Campbell and Shiller, 1988; Campbell et al., 2006; Brunnermeier and Julliard, 2008; Ambrose et al., 2013). The innovation in our paper lies in its focus on the long-run trends of house price and rent in a non-stationary environment with changing fundamental variables such as income, population, and land supply.

Our emphasis on the changing demographics also links this paper to Mankiw and Weil (1989) and voluminous ensuing empirical studies on the impact of demographics on house price. An important difference is that our model endogenizes housing supply and provides useful counterfactual exercises.

Finally, our modelling strategy resembles Kiyotaki et al. (2011) in that we have a representative firm that issues equity to finance land purchase and new capital in order to produce houses. Rather than studying the perturbation of the economy around the balanced growth path as in Kiyotaki et al. (2011), we focus on house price and rent dynamics during the economic transition phase.

3 Model

In our model economy, there exist overlapping generations of households and a long-lived representative firm that produces houses. In each period, households choose between renting and owning; conditional on the tenure choice, households choose optimal housing and non-housing consumption as well as investments. For renters, housing serves strictly as consumption. For homeowners, housing serves both as consumption and as an investment. The investment role of housing comes from the fact that homeowners collect the capital gains or losses on every unit of housing they owned, as well as rental income from additional housing units that they owned purely for investment purpose. Households face idiosyncratic income shocks and medical expense shocks. They also differ in age and initial wealth. The housing firm issues equity to finance land purchases and capital investment in order to build the optimal number of housing units that maximizes the firm value.

There are two financial assets that a household can invest savings into – a risk-free asset and a housing equity.[10] The risk-free rate of return is exogenous, but the return on housing

[10]Our model excludes stocks because "bank deposit accounts are the predominant investment vehicle in China" and that stock investment in China is "still small by size relative to the pool of savings and has not offered attractive returns in the last two decades" (Fang et al., 2016). According to the 2008 Survey of Chinese Consumer Finance and Investor Education, only 32 percent of Chinese households have any kind of stocks, bonds or mutual funds.

equity is endogenously determined in the equilibrium. There also exists a mortgage market with an exogenous mortgage interest rate.

The dynamics of macroeconomic fundamentals are exogenously specified and common knowledge. The important housing fundamentals are aggregate income, population size, age structure and land supply. Although land supply is exogenously determined by the government, which sells some new land to the market in each period, land prices are endogenously determined to clear the land market. There exists no aggregate uncertainty in this economy and no productivity shock. Consequently house price is non-stochastic.[11]

3.1 Firm

The representative firm maximizes the shareholder value. It combines land and capital to produce houses. The production function and dynamic optimization problem are laid out below.

3.1.1 Production Function

Letting K and L denote capital and land input, the firm's production function is

$$H_t = Z L_t^{\theta} K_t^{1-\theta}, \tag{1}$$

where Z is a scaling parameter, and $\theta \in (0, 1)$ measures the relative importance of land in housing construction. As in Kiyotaki et al. (2011), our specification assumes that the firm can continuously adjust housing production.[12]

We abstract away from labor input in housing construction for simplicity and transparency. In an extended version of the model (available upon request), we include labor

[11] The impact of aggregate uncertainty on housing demand in general depends on the nature and extent of uncertainty. For example, Han (2013) shows that house price uncertainty may reduce housing demand by adding capital gain risk or increase housing demand by strengthening the hedging role of housing. See also Hendershott and Hu (1981), Summers (1981), Goodwin (1986) and others. Ultimately the net effect of aggregate uncertainty on housing demand is an empirical question, which depends on the local regulations, institutional arrangements and the availability of alternative financial instruments.

[12] Here we adopt the zero adjustment cost assumption, which is common in models that build upon the neoclassical growth theory. Under this assumption, our production function is identical to an alternative specification where the developer firm produces new housing units each period with newly supplied land and capital. In reality, downward adjustment of housing stock is difficult, at least in the short run. However, we focus in this paper on a growing economy with no productivity shocks, hence downward adjustment never happens.

input in the model and find its impact on equilibrium house price for Beijing to be minor. This is consistent with the empirical finding in the previous studies that land price is much more important than labor cost in determining the house price for big cities.[13]

3.1.2 Timing and Flow of Funds

At the start of period t, the firm already owns H_{t-1} units of housing produced using K_{t-1} units of capital and L_{t-1} units of land. Without loss of generality, we normalize the number of shares in housing firm equity to be the same as the number of housing units. At the beginning of period t, denote per unit house price (also price per share of housing equity) as p_{t-1} and per unit rent (also dividend per share) as r_t. The firm collects rental income $r_t H_{t-1}$ and pays it out to shareholders as dividends. Then the firm issues new shares to raise capital and purchase land for the construction of new housing. At the end of period t (after the issuance of new shares), the number of total housing units becomes H_t, and the house price becomes p_t.

The firm's flow of funds in period t is

$$p_t(H_t - H_{t-1}) = K_t - (1-\delta)K_{t-1} + q_t(L_t - L_{t-1}), \tag{2}$$

where δ is the depreciation rate of capital, and q_t is land price in period t. The left side of the equation above represents the proceeds from issuing new shares, which are used to purchase additional capital and land, as shown on the right hand side of (2).

3.1.3 Optimization Problem

At the beginning of period t, the firm decides on the purchase of new capital and land to maximize value for existing shareholders after the issuance of new shares, $p_t H_{t-1}$. From equation (2), we have

$$\begin{aligned}
p_t H_{t-1} &= p_t H_t - [K_t - (1-\delta)K_{t-1}] - q_t(L_t - L_{t-1}) \\
&= \frac{r_{t+1} + p_{t+1}}{(r_{t+1} + p_{t+1})/p_t} H_t - [K_t - [1-\delta)K_{t-1}] - q_t(L_t - L_{t-1}) \\
&= \frac{1}{R_{t+1}}(r_{t+1}H_t + p_{t+1}H_t) - [K_t - (1-\delta)K_{t-1}] - q_t(L_t - L_{t-1}) \\
&= \frac{r_{t+1}}{R_{t+1}} H_t + \frac{1}{R_{t+1}} p_{t+1} H_t - [K_t - (1-\delta)K_{t-1}] - q_t(L_t - L_{t-1}) \\
&= \tilde{r}_t H_t - [K_t - (1-\delta)K_{t-1}] - q_t(L_t - L_{t-1}) + \frac{1}{R_{t+1}} p_{t+1} H_t
\end{aligned} \tag{3}$$

[13] See Davis and Heathcote (2007) for evidence on the US market and Wu et al. (2012) for evidence on the Chinese market.

where R_{t+1} is the total return on housing equity between t and $t+1$, defined as

$$R_{t+1} = \frac{r_{t+1} + p_{t+1}}{p_t}. \tag{4}$$

Note that $R_{t+1} - 1$ is also the cost of financing for the firm. We define $\tilde{r}_t = \frac{r_{t+1}}{R_{t+1}}$, so that $\tilde{r}_t H_t$ is the present value at t of rental income to be collected at the beginning of next period.

Therefore firm's value and optimal decisions regarding the new stocks of capital and land depend on the stocks of capital and land carried over from the previous period, denoted K_{t-1} and L_{t-1} respectively. In the dynamic programming problem of the firm, the state vector is (K_{t-1}, L_{t-1}). Using $V(K_{t-1}, L_{t-1})$ to denote the value of the firm given the state vector, the firm's optimization is

$$\begin{aligned} \max_{K_t, L_t} \quad & \tilde{r}_t H_t - [K_t - (1-\delta)K_{t-1}] - q_t(L_t - L_{t-1}) + \frac{1}{R_{t+1}} V(K_t, L_t) \\ s.t. \quad & H_t = Z K_t^{1-\theta} L_t^{\theta}. \end{aligned} \tag{5}$$

First order conditions with respect to K_t and L_t are

$$Z(1-\theta)\tilde{r}_t \left(\frac{K_t}{L_t}\right)^{-\theta} = 1 - \frac{1}{R_{t+1}} \frac{\partial V(K_t, L_t)}{\partial K_t} = 1 - \frac{1-\delta}{R_{t+1}} \tag{6}$$

$$Z\theta\tilde{r}_t \left(\frac{K_t}{L_t}\right)^{1-\theta} = q_t - \frac{1}{R_{t+1}} \frac{\partial V(K_t, L_t)}{\partial L_t} = q_t - \frac{q_{t+1}}{R_{t+1}} \tag{7}$$

In the last equality of (6) and (7), we have used the envelope conditions: $\frac{\partial V(K_t,L_t)}{\partial K_t} = 1 - \delta$ and $\frac{\partial V(K_t,L_t)}{\partial L_t} = q_{t+1}$.

From equation (6), capital input relative to land, $\frac{K_t}{L_t}$, decreases with R_{t+1}, because higher R_{t+1} means a higher cost of capital. On the other hand, equation (7) shows that $\frac{K_t}{L_t}$ increases with the cost of land. The firm can acquire land at the price of q_t in period t, and then sell it for q_{t+1} in the next period. Discounting land price in period $t+1$ to period t using R_{t+1}, the cost of one unit of land is thus $q_t - q_{t+1}/R_{t+1}$. Intuitively, higher land price induces substitution of capital for land, leading to a higher $\frac{K_t}{L_t}$.

3.1.4 Optimal Housing Supply

In this subsection, we first derive the optimal housing supply as a function of the exogenous land supply in period t denoted by L_t^*. Then we determine the market clearing land price by adjusting the land price so that land demand by the firm equals the exogenous land supply.

The optimal level of capital given land supply L_t^*, house rent r_t and financing cost R_{t+1} can be obtained from the first order condition (6):

$$K_t^* = \left[\frac{Z(1-\theta)\tilde{r}_t}{1-(1-\delta)/R_{t+1}}\right]^{1/\theta} L_t^* \tag{8}$$

Plugging this expression into the housing production equation (1), we derive the following housing supply function:

$$H_t = Z^{1/\theta} \left[\frac{(1-\theta)\tilde{r}_t}{1-(1-\delta)/R_{t+1}} \right]^{(1-\theta)/\theta} L_t^* \tag{9}$$

Thus in our model, housing supply depends critically on land supply. When more land is supplied by the government, the firm optimally chooses more capital investment (equation (8)), and hence housing supply is increased.

From equation (9), everything else being equal, housing supply decreases with housing equity return (which is also the cost of financing for the housing firm) and increases with the rent. Thus, when there is excessive investment demand, price p_t increases and R_{t+1} falls, so that supply rises; when housing consumption demand exceeds housing supply, rent r_t increases so that supply rises. This helps us design the algorithm that searches numerically for the market clearing paths of house price and rent.

3.1.5 Market Clearing Land Price

Using the land market equilibrium condition that the firm uses L_t^* unit of land, equation (7) becomes

$$\theta \tilde{r}_t H_t = \left(q_t - \frac{q_{t+1}}{R_{t+1}} \right) L_t^*. \tag{10}$$

Recall that θ is the land share in the Cobb-Douglas housing production function, and $\tilde{r}_t H_t$ is the firm's revenue in period t. The left side of the equation (10) is the share of revenue attributed to land as one of the production factors, while the right side is the cost of land.

Equation (10) can be rewritten into the following relation which can be used to solve land price recursively:

$$q_t = \frac{\theta \tilde{r}_t H_t}{L_t^*} + \frac{q_{t+1}}{R_{t+1}} \tag{11}$$

Intuitively, current land price q_t equals the land share of the firm's revenue (per unit of land), plus the discounted future land price. We will show that the economy has a BGP in which land price grows at a constant factor of G_q, and the firms financing cost is a constant, denoted R_{BGP}. Therefore in the BGP, land price can be expressed as a function of G_q, R_{BGP}, and rent.

To solve for the land price in the BGP, we obtain the following dynamic relation for land prices from equations (9), (10), and (11)

$$q_t - \frac{q_{t+1}}{R_{t+1}} = \theta Z^{1/\theta} \tilde{r}_t^{1/\theta} \left[\frac{1-\theta}{1-(1-\delta)/R_{t+1}} \right]^{(1-\theta)/\theta} \tag{12}$$

In the BGP, $q_{t+1} = q_t G_q$ and $R_{t+1} = R_{BGP}$, thus

$$q_t = M \frac{\tilde{r}_t^{1/\theta}}{1 - G_q/R_{BGP}}, \tag{13}$$

where $M = \theta Z^{1/\theta} \left[\frac{1-\theta}{1-(1-\delta)/R_{BGP}}\right]^{(1-\theta)/\theta}$ is a function of R_{BGP}, a constant denoting the firm's financing cost in the BGP.

In the ***Proposition*** we will show that rent r_t grows at a constant factor G_r in the BGP. Moreover, $G_q = G_r^{1/\theta}$. Plugging this into equation (13), we have:

$$\begin{aligned}
q_t &= M \frac{\tilde{r}_t^{1/\theta}}{1 - G_r^{1/\theta}/R_{BGP}} \\
&= M \tilde{r}_t^{1/\theta} \left[1 + \frac{G_r^{1/\theta}}{R_{BGP}} + \left(\frac{G_r^{1/\theta}}{R_{BGP}}\right)^2 + \left(\frac{G_r^{1/\theta}}{R_{BGP}}\right)^3 ...\right] \\
&= M \left[\tilde{r}_t^{1/\theta} + \frac{(\tilde{r}G_r)^{1/\theta}}{R_{BGP}} + \frac{(\tilde{r}G_r^2)^{1/\theta}}{R_{BGP}^2} + \frac{(\tilde{r}G_r^3)^{1/\theta}}{R_{BGP}^3} ...\right] \\
&= M \left[\tilde{r}_t^{1/\theta} + \frac{\tilde{r}_{t+1}^{1/\theta}}{R_{BGP}} + \frac{\tilde{r}_{t+2}^{1/\theta}}{R_{BGP}^2} + \frac{\tilde{r}_{t+3}^{1/\theta}}{R_{BGP}^3} ...\right]
\end{aligned} \tag{14}$$

That is, in the BGP, land price q_t is the sum of discounted rental rates from period t on raised to the power of $1/\theta$.

Denote by r_{BGP} and q_{BGP} the housing rent and the land price respectively at the time the economy reaches the BGP, then

$$q_{BGP} = M \frac{r_{BGP}^{1/\theta}}{1 - G_r^{1/\theta}/R_{BGP}} \tag{15}$$

In the quantitative analysis that follows, we first derive q_{BGP} from r_{BGP} and R_{BGP} which in turn are obtained from a set of regularity conditions. Then we compute equilibrium paths of house price and rent via backward induction, and back out the land price path during the economic transition based on equation (12). This is the major step in solving our model numerically.

3.2 Households

The economy is populated by a growing mass of households. A household works starting at age J_0 and retires at J_1, then lives up to a maximum age of J. In the numerical analysis, $J_0 = 21$, $J_1 = 60$ and $J = 96$. The households have homogenous preferences and beliefs, but they are heterogeneous in their age and initial wealth. In each age cohort, there are many individual households who are subject to idiosyncratic shocks to their income, medical expense and mortality (see Section 3.2.2 for details). These shocks generate within-cohort heterogeneity in income, consumption, savings, home ownership and housing size. In aggregate, the idiosyncratic risks average out and there is no aggregate uncertainty.

Households choose between owning and renting, therefore home ownership is an endogenous outcome in the model. They also choose housing and non-housing consumptions as well as investments to maximize life-time utility. Homeowners can invest in both the risk-free asset and housing equity, but the only vehicle of saving for the renters is the risk-free asset.

3.2.1 Utility function and bequest value

We assume the Cobb-Douglas utility for households in each period:

$$u(c,h) = \frac{\left[c^{1-\omega}h^{\omega}\right]^{1-\gamma}}{1-\gamma}, \tag{16}$$

where h is the housing consumption (the size of house one lives in, either rented or owned), c is the non-housing consumption, ω is a parameter measuring the relative importance of housing consumption in utility, and γ is the inverse of intertemporal elasticity of substitution (EIS). This utility form implies a unit elasticity of substitution between housing and nonhousing consumption for which Morris and Ortalo-Magne (2011) find strong data support. Further, it implies that in the BGP of our model, consumption, investment, house price and rent grow at constant rates.

At the end of period t, a household of age a dies with probability ν_a.[14] We assume that the value of bequeathing s_a shares of equity at age a is

$$V_b(s_a) = \max_{c,h} \quad \mathcal{B}u(c,h),$$

s.t.

$$c + r_t h = p_t s_a.$$

where parameter $\mathcal{B}$ determines the strength of the bequest motive. In other words, the deceased evaluates the utility as if the beneficiaries consume the bequeathed wealth in just one period, optimally splitting it between housing and non-housing consumption. With the Cobb-Douglas preference, the bequest value of equity shares when a homeowner dies is:

$$V_b(s_a) = \mathcal{B}\left[(1-\omega)^{1-\omega}\omega^{\omega}\right]^{1-\gamma}\left(\frac{1}{r_t}\right)^{\omega(1-\gamma)}\frac{(p_t s_a)^{1-\gamma}}{1-\gamma}. \tag{17}$$

[14] See Section 4.1.1 for details. Households face mortality risks that increase with age. Assets of the households that leave the economy are distributed evenly among the young households aged J_0.

Similar arguments show that the bequest value of b_a amount of risk-free asset when a renter dies is:

$$V_b(b_a) = \mathcal{B}\left[(1-\omega)^{1-\omega}\omega^{\omega}\right]^{1-\gamma}\left(\frac{1}{r_t}\right)^{\omega(1-\gamma)}\frac{(b_a)^{1-\gamma}}{1-\gamma}. \tag{18}$$

3.2.2 Household Income and Medical Expense

Income growth is one of the key driving forces of housing demand. Household income consists of two components, one deterministic and the other stochastic. Letting $y(i,a,t)$ be the income of the i^{th} household at age $a \leq J_1$ and year t, then

$$y(i,a,t) = \tilde{y}(i,a,t) \times \overline{y}(a,t), \forall a \leq J_1, \tag{19}$$

where $\tilde{y}(i,a,t)$ and $\overline{y}(a,t)$ are the stochastic and the deterministic components respectively. The deterministic income, $\overline{y}(a,t)$, includes an age effect (a) capturing the hump-shaped life-cycle profile of income and a time effect (t) for the growth of the aggregate income. We assume an AR(1) process for the logarithm of the stochastic component of income:

$$\ln\tilde{y}(i,a,t) = \rho_y \ln\tilde{y}(i,a-1,t-1) + \epsilon(i,a,t), \forall a \leq J_1, \tag{20}$$

where $\epsilon_{i,a,t}$ is the idiosyncratic shock to the i^{th} household in year t, and ρ_y determines the persistence of the shock. Regardless of time and the household's age, $\epsilon_{i,a,t}$ is drawn from a normal distribution with a mean of zero and a standard deviation of σ_y. For a household just entering the labor market, the age is J_0 and we assume $\tilde{y}(i,J_0,t) = \epsilon(i,J_0,t)$.

After retirement, households are no longer subject to income shocks. Post-retirement income is assumed to grow at the same rate as the aggregate income. However, retirees are faced with stochastic out-of-pocket medical expenses, denoted by $m(i,a,t)$, which is assumed to have an idiosyncratic stochastic component $\tilde{m}(i,a,t)$ and a deterministic component $\overline{m}(a,t)$ that is common for all individuals of the same age a at time t:

$$m(i,a,t) = \tilde{m}(i,a,t) \times \overline{m}(a,t), \forall a > J_1, \tag{21}$$

We assume that $\ln\tilde{m}(i,a,t)$ follows an $AR(1)$ process:

$$\ln\tilde{m}(i,a,t) = \rho_m \ln\tilde{m}(i,a-1,t-1) + \eta(i,a,t), \forall a > J_1, \tag{22}$$

where $\eta(i,a,t)$ is drawn from a normal distribution with a mean of zero and a standard deviation of σ_m. Since out-of-pocket medical expense is small relative to income before retirement, we assume it is zero for simplicity.

We include out-of-pocket medical expense shocks in the model for several reasons. First, recent studies show that stochastic medical expense is an important determinant of wealth accumulation/decumulation for retirees (De Nardi et al., 2010). Second, medical insurance is usually under-provided in emerging markets. Third, our framework can serve as a tool to examine the impact of better provision of medical insurance on housing demand and price.

3.2.3 Housing Investment and Leverage

A household enters the economy as a renter and saves via the risk-free asset before becoming a homeowner. Barriers to homeownership include the minimum down payment requirement and the minimum (starter) housing size. To buy a house of size s at price p, a household needs to pay down at least $d \times p \times s$ where d is the minimum down payment, and $s \geq \underline{s}$, the minimum housing size. Thus a household needs to save at least $d \times p \times \underline{s}$ before she becomes a homeowner.

Homeowners can use mortgages to borrow up to $1 - d$ of the house values. Thus, they hold leveraged investments in housing equity. The mortgage interest rate r_m is exogenous. There is no mortgage default in our model. Given the housing fundamentals in the model, the equilibrium housing return is certain and positive. Households that experience negative idiosyncratic income or medical expense shocks can choose smaller housing investments or exit the housing market completely. We assume mortgage adjustment, such as refinancing, is costless.

Suppose a household of age a holds $s_{a-1,t-1}$ shares of housing equity at the end of $t-1$. At the beginning of the period t, the household's non-financial income $y_{a,t}$ and out-of-pocket medical expense $m_{a,t}$ are revealed. Housing firm dividend per share r_t is also realized and paid to the exisiting shareholders, and then housing equity is traded. The household decides whether to enter the housing market and becomes a homeowner if $s_{a-1,t-1} = 0$, or change the holding of housing equity if $s_{a-1,t-1} > 0$. In some cases, the homeowner may find it optimal to exit the housing market completely and becomes a renter. For example, old households tend to have an incentive to exit the housing market despite the high housing return. This is because their non-housing consumption may be sub-optimal if they keep their housing assets above the minimize house size (which becomes too big for old households).

If the household decides to enter or stay in the housing market, she needs to decide on the quantity of housing consumption $h_{a,t}$, nonhousing consumption $c_{a,t}$ and shares of equity $s_{a,t}$. If the household decides to exit or stay out of the housing market, she needs to decide on the amount of risk-free asset holding $b_{a,t}$, as well as $h_{a,t}$, and $c_{a,t}$.

In our model, both housing and risk-free asset are safe and it is costless to adjust housing investment. Consequently it is always optimal to invest only in the asset that provides the higher return. Since house price and rent are endogenously determined in our model, given an exogenous risk-free rate, house price and rent adjust until the leveraged return on the housing investment is sufficiently high so that the demand for housing equity equals the supply of it in each period. In other words, the leveraged return on the housing asset exceeds the return on the risk-free asset in each period. Therefore, homeowners invest in housing asset only, and hold zero risk-free asset unless they exit the housing market when their income is too low to maintain the minimum housing size. Renters save in the form of the risk-free asset. After they have saved enough for the minimum down payment, they can become homeowners and invest in the housing equity.

3.2.4 Household's Optimization Problem

For ease of presentation, we omit the household index i and the time index t from the choice variables of households, but keep the age index a. For aggregate variables such as price and rent, the time index is still used.

The homeowner's problem For a homeowner of age a who enters the current period t with s_{a-1} shares of housing equity, income y_a and medical expense m_a, we use $V^{rent}(s_{a-1}, y_a, m_a)$ to denote her value if she exits the housing market and becomes a renter this period, and $V^{own}(s_{a-1}, y_a, m_a)$ to denote the her value if she stays in the housing market. The value function of a homeowner given the state vector (s_{a-1}, y_a, m_a) is

$$V(s_{a-1}, y_a, m_a) = \max\{V^{rent}(s_{a-1}, y_a, m_a), V^{own}(s_{a-1}, y_a, m_a)\}.$$

Specifically,

$$\begin{aligned} &V^{own}(s_{a-1}, y_a, m_a) = \max_{s_a, h_a} u(c_a, h_a) + \beta\mathcal{E}\left[(1-\nu_a)V(s_a, y_{a+1}, m_{a+1}) + \nu_a V_b(s_a)\right], \\ &s.t. \qquad r_t h_a + c_a = y_a - m_a + [p_t + r_t - LTV_{t-1}p_{t-1}(1+r_m)]s_{a-1} - (1-LTV_t)p_t s_a, \\ &\qquad\qquad \text{and} \quad s_a \geq \underline{s}. \end{aligned}$$

where in the budget constraint, ν_a is the probability of death by the end of the period for someone of age a, $V_b(s_a)$ is the bequest value of equity shares given in (17), and LTV_t is the loan-to-value ratio of the homeowner in period t given by

$$LTV_t \begin{cases} = 1-d, & \text{if } r_m < R_{t+1}; \\ = 0, & \text{if } r_m \geq R_{t+1}, \end{cases}$$

where r_m is the mortgage rate, and R_{t+1} is the total return on the housing equity defined in equation (4). That is, homeowners use leverage via the mortage and borrow up to the limit (i.e., only pays the minimum down payment) if the mortgage rate is lower than the housing equity return, and borrow nothing otherwise. The right hand side of the budget constraint for the homeowner takes into consideration the interest payment of the outstanding loan balance as well as the change in the loan size as the household adjusts the equity holding. Since the homeowner obtains a leveraged position in housing equity using mortgage, the actual rate of return $r^{lev}_{s,t+1}$ she receives from housing investment between t and $t+1$ is given by

$$r^{lev}_{s,t+1} = \frac{1}{1-LTV_t}(R_{t+1}-1) - \frac{LTV_t}{1-LTV_t} r_m.$$

In a similar vein, the value function of a household that enters the current period as a homeowner but switches to a renter this period is given by:

$$\begin{aligned} V^{rent}(s_{a-1}, y_a, m_a) &= \max\nolimits_{b_a,h_a} u(c_a,h_a) + \beta \mathcal{E}\left[(1-\nu_a)W(b_a, y_{a+1}, m_{a+1}) + \nu_a V_b(b_a)\right], \\ s.t. \quad & r_t h_a + c_a = y_a - m_a + [p_t + r_t - LTV_{t-1}p_{t-1}(1+r_m)]s_{a-1} - b_a, \\ & \text{and} \quad b_a > 0, \end{aligned}$$

where $W(b_a, y_{a+1}, m_{a+1})$, to be defined below, is the next period's value function of a renter given the state vector (b_a, y_{a+1}, m_{a+1}).

The renter's problem Consider a renter household of age a who enters the current period t with b_{a-1} amount of risk-free asset. In the beginning of the current period, her income y_a and out-of-pocket medical expense m_a are realized. Let $W^{rent}(b_{a-1}, y_a, m_a)$ denote her value function if she continues to rent in the current period, and $W^{own}(b_{a-1}, y_a, m_a)$ denote her value function if she enters the housing market this period. The household compares the value of renting vs. owning to decide whether to enter the housing market. Overall, the value function of a renter given the state vector (b_{a-1}, y_a, m_a) is

$$W(b_{a-1}, y_a, m_a) = \max\{W^{rent}(b_{a-1}, y_a, m_a), W^{own}(b_{a-1}, y_a, m_a)\}.$$

Specifically,

$$\begin{aligned} W^{rent}(b_{a-1}, y_a, m_a) &= \max\nolimits_{b_a,h_a} u(c_a,h_a) + \beta \mathcal{E}\left[(1-\nu_a)W(b_a, y_{a+1}, m_{a+1}) + \nu_a V_b(b_a)\right], \\ s.t. \quad & r_t h_a + c_a = y_a - m_a + (1+r_b)b_{a-1} - b_a, \\ & \text{and} \quad b_a > 0 \end{aligned}$$

where $V_b(b_a)$ is the bequest value of risk-free investment given in (18), and r_b is the exogenous risk-free rate.

$$
\begin{aligned}
W^{own}(b_{a-1}, y_a, m_a) &= \max_{s_a, h_a} u(c_a, h_a) + \beta \mathcal{E}\left[(1-\nu_a)V(s_a, y_{a+1}, m_{a+1}) + \nu_a V_b(s_a)\right], \\
s.t. \qquad & r_t h_a + c_a = y_a - m_a + (1+r_b)b_{a-1} - (1-LTV_t)p_t s_a, \\
& \text{and} \quad s_a \geq \underline{s},
\end{aligned}
$$

where $V(s_a, y_{a+1}, m_{a+1})$ is the next period's value function of a homeowner defined above.

3.2.5 Optimality Conditions

In each period, a household makes both intra-temporal and inter-temporal decisions. Inter-temporally, the household's optimal consumption-saving choices satisfy

$$
\frac{\partial u(c_a, h_{a,t})}{\partial c_a} = \beta \mathscr{R}_{t+1} \mathcal{E}\left[(1-\nu_a)\frac{\partial u(c_{a+1}, h_{a+1})}{\partial c_{a+1}} + \nu_a V_b'(w_a)\right], \tag{23}
$$

where $w_a = s_a$, $\mathscr{R}_{t+1} = 1 + r_{s,t+1}^{lev}$ for the homeowners who invest in the housing market and $w_a = b_a$, $\mathscr{R}_{t+1} = 1 + r_b$ for the renters who save with the risk-free asset. In the equation above, we have used the envelope condition that the derivative of a household's value function with respect to asset holding equals the marginal utility of consumption when evaluated at the optimal choices.

Intra-temporally, the decision is to allocate between housing and non-housing consumption. At the optimal choices, the marginal utility of housing consumption and the marginal utility of non-housing consumption satisfy the following condition:

$$
\frac{\partial u(c_a, h_{a,t})}{\partial h_a} = r_t \frac{\partial u(c_a, h_{a,t})}{\partial c_a}. \tag{24}
$$

Under the Cobb-Douglas utility fuction $u(c, h)$ in (16), this is equivalent to:

$$
\frac{c_a}{h_a} = r_t \frac{1-\omega}{\omega}. \tag{25}
$$

Equations (23) and (25), together with the household's budget constraint, determine the amount of housing and non-housing consumptions in current period and the amount of housing asset that carries over into the next period.

3.3 General Equilibrium

The general equilibrium is defined as sequences of house prices p_t, rents r_t, land prices q_t, and sequences of choices made by the firm and the households that satisfy the following conditions: (i) the firm's choices of L_t, K_t, H_t are consistent with the firm's optimization problem; (ii) the household's choices of consumptions (housing and non-housing) and investments (risk-free asset and housing equity) are consistent with the household's optimization problem; (iii) the distribution of households in the state space evolves according to the law of motion specified below; and (iv) the paths of prices and rents satisfy market clearing conditions specified below.

A number of objects need to be defined precisely before we specify the market clearing conditions and the law of motion. Households are heterogeneous in the holding of risk-free asset, holding of housing asset, age, income and medical expense, as represented by the state vector (b, s, a, y, m).[15] In each period, households are distributed in the five dimensional state space of non-negative real numbers, denoted $\mathcal{B} \times \mathcal{S} \times \mathcal{A} \times \mathcal{Y} \times \mathcal{M}$, with $b \in \mathcal{B}$, $s \in \mathcal{S}$, $a \in \mathcal{A}$, $y \in \mathcal{Y}$, $m \in \mathcal{M}$.

Let $\lambda_t(b, s, a, y, m)$ be the distribution of households in the state space at the beginning of period t. Let $B \times S \times A \times Y \times M$ be a typical subset of the state space. The probability that households with current state (b, s, a, y, m) transition into the set $B \times S \times A \times Y \times M$ at the end of period t is

$$\begin{aligned} & Q_t(B \times S \times A \times Y \times M | b, s, a, y, m) \\ = & \int_{(a', y', m') \in A \times Y \times M} \mathcal{I}\{(b', s') \in B \times S | b, s, a, y, m\} \Gamma(a', y', m' | a, y, m) da' dy' dm', \end{aligned}$$

where $\mathcal{I}$ is the indicator function, and b' and s' are the optimal decision rules regarding risk-free bond and housing equity investment solved from the household's optimization problems given the state of (b, s, a, y, m). Recall that the survival probability, the transition of stochastic income and the transition of stochastic medical expenses over time are all exogenous. Therefore we use $\Gamma(a', y', m' | a, y, m)$ to denote the exogenous probability of the transition from state (a, y, m) to state (a', y', m'), where $a' = a + 1$ if the household survives.

Law of Motion of the Distribution of Households For the local residents, the law of motion of the distribution of households in the whole state space (the product of the exogenous space $\mathcal{A} \times \mathcal{Y} \times \mathcal{M}$ and the endogenous space $\mathcal{B} \times \mathcal{S}$) is as follows:

$$\begin{aligned} & \lambda_{t+1}(B \times S \times A \times Y \times M) = \\ & \int_{(b,s,a,y,m) \in \mathcal{B} \times \mathcal{S} \times \mathcal{A} \times \mathcal{Y} \times \mathcal{M}} Q_t(B \times S \times A \times Y \times M | b, s, a, y, m) d\lambda_t(b, s, a, y, m) \end{aligned} \tag{26}$$

[15]This is a general state vector for both renters and owners. In Section 3.2 we have introduced state vectors for homeowners and renters individually conditional on their age.

For migrants of each age cohort, we assume that their distribution in the space $\mathcal{B} \times \mathcal{S} \times \mathcal{Y} \times \mathcal{M}$ is identical to the distribution of local residents of the same age, and the distribution of these local residents evolves according to equation (26). In the beginning of the next period, the migrants are included as part of the local residents.

Market Clearing Conditions In period t, the housing consumption market clears if:

$$H_t = \int_{(b,s,a,y,m) \in \mathcal{B} \times \mathcal{S} \times \mathcal{A} \times \mathcal{Y} \times \mathcal{M}} h'(b, s, a, y, m) d\lambda_t(b, s, a, y, m) \tag{27}$$

where $h'(b, s, a, y, m)$ is the optimal housing consumption given the state (b, s, a, y, m).

The equity market clears if:

$$H_t = \int_{(b,s,a,y,m) \in \mathcal{B} \times \mathcal{S} \times \mathcal{A} \times \mathcal{Y} \times \mathcal{M}} s'(b, s, a, y, m) d\lambda_t(b, s, a, y, m) \tag{28}$$

where $s'(b, s, a, y, m)$ is the optimal housing investment given the state (b, s, a, y, m). The aggregate outcome of all housing investment demand must equal the supply which is denoted by H_t.

Finally, the land market clearing condition is:

$$L_t = L_t^*$$

where L_t is the firm's land demand and L_t^* is the exogenous land supply in period t.

3.4 Balanced Growth Path

Assume that from year T_{BGP} forward, aggregate income, land supply and population grow at fixed factors G_Y, G_L and G_N respectively. In addition, age distribution of population no longer changes over time. Then we have the following proposition (with proof provided in Appendix A) about the properties of the BGP.

Proposition *A BGP exists and is characterized by the following:*

1. *Aggregate capital growing at a factor of* $G_K = G_Y$*;*
2. *Aggregate housing supply growing at a factor of* $G_H = G_Y^{1-\theta} G_L^{\theta}$*;*
3. *Housing investment demand and consumption per capita growing at* $G_s = (G_Y/G_N)^{1-\theta}(G_L/G_N)^{\theta}$ *and* $G_h = G_s$*;*
4. *Demand for risk-free asset per capita growing at a factor of* G_Y*;*

5. *Non-housing consumption per capita growing at* $G_c = G_Y/G_N$*;*

6. *House price growing at* $G_p = (G_Y/G_L)^{\theta}$*;*

7. *House rent growing at* $G_r = (G_Y/G_L)^{\theta}$*;*

8. *Land price growing at* $G_q = G_Y/G_L$*;*

9. *Floor-area ratio, defined as* H/L*, growing at* $G_{FAR} = (G_Y/G_L)^{1-\theta}$*;*

10. *Constant price-income ratio (*pH/Y*) and price-rent ratio (*p/r*).*

Equilibrium in the BGP has a set of properties that are consistent with stylized facts. For example, house price is driven by income growth and land supply. The importance of income is shown in Case and Shiller (2003), and the importance of land supply is emphasized in Glaeser et al. (2005) and Saiz (2010). It has been observed that house price indices, after controlling for inflation, can exhibit extremely low growth rates in the long run.[16] This phenomenon can be generated in our model when land supply and aggregate income grow at similar rates. On the other hand, when land supply grows at a lower rate than income, the model predicts a growing trend of house price, consistent with the pattern of house price experienced in the past decades by cities such as Hong Kong, San Francisco and New York.

By comparing the sixth and the eighth points in the ***Proposition***, we can see that the growth rate of land price is always higher than that of house price since $\theta < 1$. This is consistent with the empirical observations of major cities in both the U.S. (Davis and Heathcote (2007)) and China (Deng et al. (2012)). In addition, price-income ratio and price-rent ratio are both constants in the BGP. This is consistent with Ambrose et al. (2013) who document that price-rent ratio exhibits long-run stationarity.

It should be noted that the economy does not operate in the BGP immediately after the stabilization of the exogenous variables. It needs to wait until the age distribution and the asset distribution of households become time-invariant. In the quantitative analysis below, we assume that after year 2044, all exogenous variables grow at constant rates. After another 70 years, i.e., after 2114, the age distribution and asset distribution will be time-invariant. Therefore, the Beijing housing market enters into BGP in 2114 under our model.

The key variables grow at constant rates in the BGP, therefore they can be re-scaled so the the economy operates as if it is in a steady state. In the quantitative analysis, we start with finding the house price and rent in this "steady state"; then we use them as the

[16]See, e.g., Chart 4 in Shiller (2007).

terminal conditions to solve for the equilibrium paths of price and rent during the transition periods using backward inductions.

4 Quantitative Analysis: Projection and Calibration

Having studied the analytical features of our model economy in the BGP, we now turn to the transition periods where the non-stationarity of fundamental variables is important for the determination of optimal demand and supply in the housing market. Since there are no aggregate uncertainties in the model, the equilibrium prices (e.g. house price, rent and land price) and quantities (e.g. housing supply) all follow deterministic paths, which are taken into considerations by all market participants (the housing firm and households) in their rational, forward-looking decision-making. Because of uninsurable idiosyncratic income and medical expense shocks, the model features an incomplete market, and does not admit of an analytical solution along the transition path. We develop an efficient method to solve the dynamic equilibrium numerically (see Appendix B for details) using backward induction with the properties of the BGP as the terminal conditions.

It is important to note that in solving the model equilibrium during the economic transition periods, we do not rely on any historical data for the Beijing housing market (such as recent housing prices). We only use historical data about fundamental variables in Beijing to estimate their dynamics and then project their future evolutions. In addition, we need market data from a reference city for the terminal conditions. In the baseline specification, we consider Hong Kong, a well-developed housing market, as a good reference for Beijing when it enters the BGP. These two cities have similar cultural background, preferences over housing, and risk tolerance. They both adopt a land lease policy which has been shown to have a significant impact on housing price dynamics (Anglin et al., 2014).[17] One legitimate concern is that the two cities are connected in their economies and differ in the land expansion possibilities. To address this, we present an alternative specification using Washington, D.C. as a reference city (see Section 5.3.9).

We calibrate the model parameters so that several key properties of the Beijing housing market in the BGP resemble those computed based on recent data from Hong Kong. Given the projected exogenous fundamentals and a set of calibrated model parameters, we endoge-

[17]Land lease affects house price and rent through the uncertainties associated with the renewal of land lease: renewal is not guaranteed and the cost of renewal is not certain. Such uncertainties not just affect house price directly, but also indirectly through altering the timing and frequency of home redevelopment. It is beyond the scope of our paper to explicitly model the effects of the land lease policy.

nously solve the paths of house price, rent, and land price that clear the housing investment market, the housing consumption market and the land market.

An alternative approach is to first use historical data from the Beijing housing market to fit some reduced-form model of housing price as a function of the fundamentals and then extrapolate future house price based on the estimated relationship and the projection of the fundamentals. But this approach is not suitable for a transition economy where the relationship between house price and fundamentals is complex and unstable (see Section 5.2 for more details). In contrast, our approach is based on a dynamic equilibrium model without relying on historical house price data. Therefore, our approach not only circumvents non-stationarity encountered in a transition economy, but also shed light on how the equilibrium prices change in response to the changes in the fundamental variables.

4.1 Projections of Population, Income, and Land Supply

This subsection explains in details the evolutions of aggregate economic fundamentals, the initial conditions of households and the specifications of the idiosyncratic risks they face, as well as the calibration of model parameters. In the quantitative analysis below, we set the initial date T_0 to be 2005, which is the start of the rapid growth of house price in Beijing.

4.1.1 Evolution of Population Structure

Two dimensions of the population structure are relevant for our model: population size and age distribution of the population. Population size directly affects housing demand. Age distribution matters because housing demand is age-specific.

The population data are obtained from the 2010 Census, and from *Sample Survey on Population Change* for other years between 2005 and 2013. The upper-left panel of Figure 1 plots the age distribution of Beijing residents, defined as individuals who either have formal registration (Hukou) or live in Beijing for more than six months per year. Compared with the overall urban population, Beijing population is much younger due to the influx of young migrants.

To project the population structure after 2013, we need to predict the fertility rate, the mortality rate and the immigration rate of the Beijing population. Given the recent relaxation of the one-child policy in China, we assume that the fertility rate switches from an "old rate" regime to a "new rate" regime in 2024. For the "old rate" regime, we use the data between 2005 and 2013 to calculate the age-specific fertility rate and mortality rate of Beijing population. If this trend continues, it would imply an ever decreasing total population in the future. For the “new rate” regime, we assume that beginning in 2024, the fertility trend rises linearly for the next 10 years so that the overall population growth rate reaches 0.4% in 2034 and then fertility rate is assumed to be time-invariant afterwards. Both trends are presented in the upper-right panel of Figure 1. Given that households have perfect foresight, the switch of the regimes is taken with full certainty. The mortality rate is assumed to be constant over time since life expectancy in China is already close to that of industrialized countries.

As emphasized in Henderson (2010), rapid urbanization is one of the key issues in population dynamics for a transition economy. Urbanization is reflected in the migration of households to Beijing in our model. At the end of 2014, Beijing permanent residents are comprised of 13.5 million local residents with Hukou and another 8.02 million “resident alien population” without Hukou – migrants who move from elsewhere in China (rural or other urban cities) and live in Beijing for more than six months per year. The migration rate, or the number of new migrants as a fraction of existing Beijing population, has been declining since 2008, and averages around 2.88% between 2010 and 2014.[18] In the baseline model, we assume that the migration rate decreases linearly from 2.88% in 2014 to zero by 2044. In the data it is clear that migrants to Beijing are mainly young workers; therefore we assume that only those aged between 21 and 30 migrate to Beijing in each year.

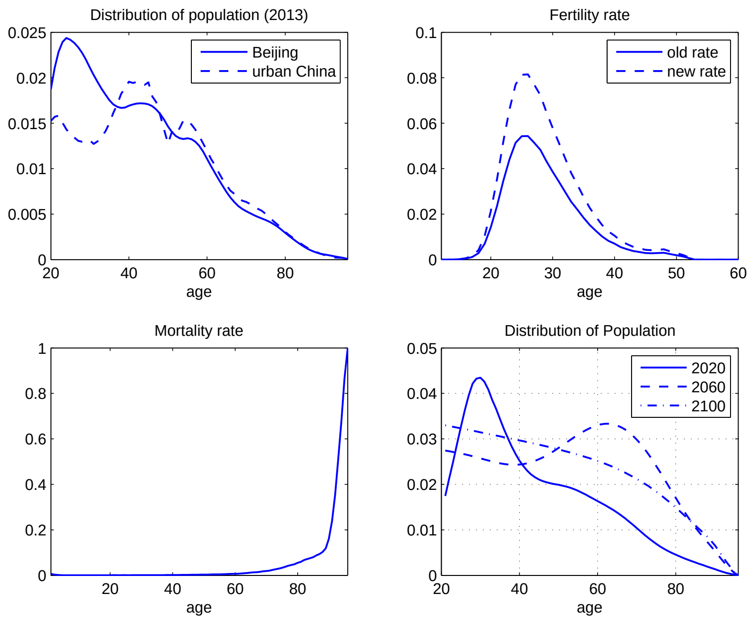
Based on the 2013 data and using the fertility rate, mortality rate and immigration rate discussed above, we extrapolate the population structure after 2013. The lower-right panel of Figure 1 plots the projected age structure of population in 2020, 2060 and 2100. Upon the completion of urbanization which is represented by a zero immigration rate after 2044, the peak age of the population moves to 65 in 2060. In year 2100, the population structure stabilizes to a profile that decreases with age, due to the increasing age-profile of the mortality rate.

4.1.2 Evolution of Land Supply

In China, local governments own land and auction land use right. The amount of land to be auctioned depends on a multitude of considerations, including policies from the central government, the fiscal balance of the local governments and the growth rate of the local GDP. Our model takes land supply as exogenously given, but land price and housing supply

[18]According to the Beijing Municipal Bureau of Statistics, Beijing has another estimated 7.5 million floating population at the end of 2013. Given the short-term nature of these migrants, we leave them out in the calculation of migration rate.

Figure 1: Fertility rate, mortality rate, and population structure



This figure shows the initial age distribution of population, fertility rate, mortality rate and projected population structure. The fertility rate is presented as one half of women's fertility rate in the data, interpreted as fertility rate per couple. The "Old Rate" is the average fertility rate between 2005 and 2013 in the data. The "New Rate" is the projected fertility rate after 2034.

are both determined endogenously.

We obtain data on the supply of residential land from the National Bureau of Statistics (NBS). For each of the major cities in China, NBS reports the amount of new residential land acquired by housing developers.[19] This is the flow of land. The stock of land in 2009 is available from the 2010 *China Statistical Year Book of Environment* complied by NBS. Table 11-3 of the year book is "Basic Statistics on Urban Area and Land Used for Construction by Region" which reports that the area of residential land was 38,330 hectares in Beijing at the end of 2009. Based on the stock of land in 2009 and the annual flows, we obtain the historical total stock of residential land in Beijing, and then divide it by the population of Beijing residents to obtain land supply per capita. From Table 1, it is clear that in Beijing the total land stock growth has significantly lagged the growth of population since 2005, leading to a declining land supply per capita.

We assume that the land supply grows at a constant rate of 0.5% from 2014 onwards in

[19]`http://data.stats.gov.cn/workspace/index?m=csnd`.

Table 1: Land and population of urban Beijing

-

	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
Population (million)	13.9	14.2	14.6	14.9	15.4	16.0	16.8	17.7	18.6	19.6	20.2	20.7	21.4
Growth rate (%)	–	2.75	2.33	2.49	3.03	4.10	4.68	5.67	5.03	5.48	2.89	2.51	3.61
New land (ha)	1472	2093	1391	1572	774	295	392	823	625	859	507	306	906
Total land stock (ha)	29518	30990	33082	34474	36046	36820	37115	37507	38330	38955	39814	40321	40627
Growth rate (%)	–	4.99	6.75	4.21	4.56	2.15	0.80	1.05	2.20	1.63	2.20	1.27	0.76
Land per capita (m^2)	21.31	21.77	22.72	23.09	23.44	23.00	22.14	21.18	20.61	19.86	19.72	19.49	18.95
Growth rate (%)	–	2.18	4.32	1.67	1.48	-1.87	-3.71	-4.37	-2.69	-3.65	-0.67	-1.21	-2.75

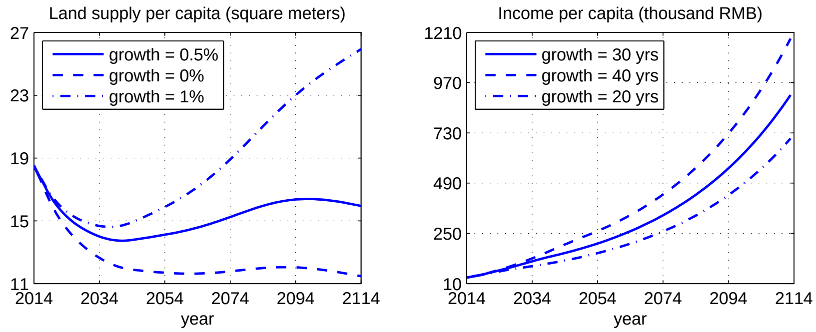
the baseline model. The evolution of land per capita is given by the solid line in the left panel of Figure 2. Land supply per capita falls gradually due to the inflow of migrants during the urbanization process. As the growth of population plateaus, land supply per capita becomes time-invariant. The two broken lines show the evolution of land per capita when the growth of aggregate land supply is either 1% or 0%, both of which will be used in the sensitivity analyses in Section 5.3.

4.1.3 Evolution of Aggregate Income

The average disposable income of Beijing residents, as reported by the NBS, is 38.17, 41.13, and 43.91 thousand Renminbi (RMB) in year 2012, 2013 and 2014 respectively, each in terms of 2014 RMB.[20] These are roughly one-third of the disposable income of Hong Kong residents in the corresponding years. We assume that average income grows at a constant rate of 7% in 2015, then the growth rate declines linearly from 7% to 3% over the next 30 years. Given the assumption that population grows at a constant rate of 0.4% in the BGP, the growth rate of average income is $(1 + 3\%)/(1 + 0.4\%) - 1 = 2.59\%$ per year. In the sensitivity analysis, we consider two alternative cases where income growth plateaus to 3% after either 20 or 40 years. Figure 2 plots the projected evolution of the per capita income under the baseline and the two alternative scenarios.

[20] Each year the NBS reports income based on a random sample of households in Beijing. The sample includes two major components: residents with Beijing Hukou and residents without Beijing Hukou but living in Beijing for over 6 months every year.

Figure 2: Projected land supply and income per capita



This figure shows the projected land supply and income under different assumptions. In the left panel, "growth" refers to the growth rate of land per capita. In the right panel, "growth" refers to number of years it takes before the growth rate of aggregate income plateaus. In the baseline mode we use "growth = 0.5%" and "growth = 30 yrs"

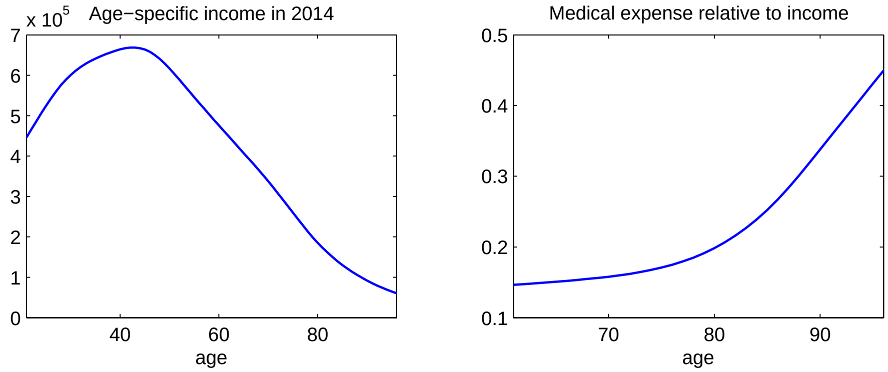
4.2 Other Exogenous Inputs

4.2.1 Initial Assets

The initial distribution of households' asset by age is an important input in the model. The best survey data in China about household assets is the China Household Finance Survey (CHFS `http://www.chfsdata.org/`). Thus far there is only one wave of data available publicly – the 2012 wave. From the 2012 survey, we estimate the ratio of financial wealth to income and the age profiles of financial asset and housing equity for urban households in China. The estimated ratio of financial wealth to income is 3.68.

Data from the Beijing Bureau of Statistics show that the disposable income per capita in 2005 is 20.02 thousand (in terms of 2014 RMB). Therefore in 2005, the estimated average financial wealth for Beijing residents is 73.65 thousand RMB. In addition, the average housing size for Beijing residents in 2005 is 19.5 square meters. We distribute these assets across different age cohorts, assuming the same age profiles of financial wealth and housing equity as in the 2012 wave of CHFS. These are the initial assets of households who enter the economy at T_0 (i.e., in year 2005). For those who enter at $t > T_0$, their initial assets are bequests from those who die in period $t-1$. The total amount of bequest wealth is endogenously determined in the model and distributed evenly among those new households just entering the economy.

Figure 3: Age profile of income and medical expense



4.2.2 Income and Medical Expense

We estimate the age profile of income, i.e., the term $\overline{y}(a,t)$ in equation (19), using panel data from China Health and Nutrition Survey (`http://www.cpc.unc.edu/projects/china/`). We use the publicly available waves of survey from 1989 to 2011. We regress the logarithm of income on age and year dummies. The coefficients on the age dummies provide us the age profile of income. The left panel of Figure 3 plots the smoothed age-profile of income, re-scaled to match the average income of Beijing households in 2014. In addition to the age profile of income, our numerical analysis takes into account the growth trend of income over time.

The residuals from the regression of the logarithm of income on age and year dummies are the logarithm of the stochastic component of household income $\ln \tilde{y}(i,a,t)$. We fit an $AR(1)$ model for $\ln \tilde{y}(i,a,t)$ as given in equation (20). The estimated parameter values are $\rho_y = 0.864$, and $\sigma_y = 0.253$.

Similarly, we use the same two-step regressions and data from the China Health and Retirement Longitudinal Study (`http://charls.ccer.edu.cn/en`) to estimate the age profile of the deterministic component $\overline{m}(a,t)$ of the medical expense as well as the $AR(1)$ process of the idiosyncratic stochastic component $\tilde{m}(i,a,t)$ given in equation (22). As shown in the right panel of Figure 3, the ratio of medical expense to income is 0.15 at age 61, and it reaches 0.45 at age 96. The estimated parameter values for the process of idiosyncratic medical expense are $\rho_m = 0.922$, and $\sigma_m = 0.498$.

4.3 Model Parameters

Our model has three parameters related to the housing production (Z, θ, δ) and four preference parameters $(\gamma, \mathcal{B}, \beta, \omega)$. We pin down these parameters by calibrating our model to match some key features of the Beijing market when it reaches the BGP. In addition, we choose the values for minimum down payment d, minimum housing size $\underline{s}$, risk-free saving rate r_b, and mortgage interest rate r_m to match the empirical observations.

In the baseline specification, we assume that Beijing housing market in the BGP will resemble the current state of Hong Kong in terms of price-income ratio, price-rent ratio and growth rate of real house price. The Hong Kong Rating and Valuation Department reports data for different housing classes in the three main regions of Hong Kong – Hong Kong island, Kowloon and New Territory. Averaging over different classes in the three regions, the annual housing rent in Hong Kong is about 3.39 thousand HK dollars per square meter in 2012. Using an exchange rate of 0.82 RMB per HK dollar in 2012, the annual rent is about 2.78 thousand RMB per square meter. The average house price is 121 thousand HK dollars in 2012 which is about 99 thousand RMB.

Price-rent ratio is calculated as *(average house price) / (average rent)* $= 99/2.78 \approx 35.6$. We validate this ratio by calculating the ratio for each class of housing in each of the three regions in 2012, then taking the average of these ratios. The resulting number is 34.8. Price-income ratio is calculated as *(price per square meter)*$\times$*(average number of square meters per capita)/(average income per capita)*. Based on the 2012 data, housing per capita is about 12 square meters and average disposable income per capita is around 100 thousand RMB in Hong Kong. Therefore the price-income ratio is 11.88.

The growth rate of real house price in Hong Kong is calculated from house price index and CPI; the former is available from the Hong Kong Rating and Valuation Department, and the latter is available from the Census and Statistics Department. During the period of 1981-2012, the geometric mean of the growth rate of house price in Hong Kong, adjusted for inflation, is 2.14%. Thus, we take the growth factor of house price for Beijing in the BGP to be $G_p = 1.0214$.

Real housing return is calculated from the price-rent ratio and the real appreciation of house price. Given the price-rent ratio of 35.6 and the real appreciation rate of house price of 2.14%, return on housing investment is $R = 1.0495$, which we take as the return on housing investment for Beijing in the BGP.

4.3.1 Parameters Related to Housing Production

Three parameters are related to housing production: land share in production (θ), capital depreciation rate (δ) and scaling parameter in housing production (Z). We pin down θ by comparing the growth rate of house price to the growth rate of land price. As is evident from the ***Proposition***, in the BGP, the growth factors of house price (G_p) and land price (G_q) satisfy $G_p = G_q^\theta$, therefore

$$\theta = \frac{logG_p}{logG_q} \tag{29}$$

We calculate the growth factors based on Hong Kong data between 1987 and 2012. During this period, θ implied by the growth rates of house price and land price is 0.839, 0.497 and 0.819 for Hong Kong island, Kowloon and New Territory respectively. We take the average value $\theta = 0.72$.

In our model, the depreciation of housing stock is captured by the depreciation of capital (δ). Leigh (1980) estimates that the annual depreciation rate of housing in the United States is between 0.0036 and 0.0136. Letting δ^h denote this housing depreciation rate, then capital depreciation rate is $(1-\delta^h)^{1/(1-\theta)}$ based on our housing production function. Therefore capital depreciation should be between 0.01 and 0.03. We use $\delta = 0.02$ in all quantitative analyses.

To pin down the scaling parameter Z, we use the housing supply equation (9). Since price-income ratio (pH/Y) and price-rent ratio (p/r) are constants in the BGP, housing supply per capita H in the BGP satisfies

$$H = \frac{ratio^{py}}{ratio^{pr}} \times \frac{Y}{r} \tag{30}$$

where $ratio^{py}$ and $ratio^{pr}$ are price-income ratio and price-rent ratio respectively, Y is the average income per capita and r is the housing rent. Substituting equation (30) into equation (9) yields

$$Z = \left(\frac{1}{r}\right)\left(\frac{ratio^{py}}{ratio^{pr}} \times \frac{Y}{L}\right)^{\theta}\left[\frac{1-(1-\delta)/R}{1-\theta}\right]^{1-\theta}. \tag{31}$$

In Hong Kong, housing per capita is about 12 square meters and the average floor-area ratio is about 4.5, thus land use per capita is $L = 12/4.5 \approx 2.67$ square meters per capita. Substituting into equation (31) the per capita income Y=100 thousand RMB, rent $r = 2.78$ thousand RMB, price-income ratio $ratio^{py} = 11.88$ and price-rent ratio $ratio^{pr} = 35.6$ as well as return on housing investment $R = 1.0495$, we obtain $Z = 1.47$.

4.3.2 House price, Rent and Land Price in the BGP

We have used information from the Hong Kong market such as per-capita income, rent, land use, and price-income and price-rent ratios to identify the parameter Z. Now we use the projected per capita income and land in Beijing to obtain house price and rent when the Beijing market enters the BGP.

From the housing supply equation (9), it is straightforward to obtain

$$\tilde{r}_t = \left(\frac{1}{Z}\right)\left(\frac{ratio^{py}}{ratio^{pr}} \times \frac{Y_t}{L_t}\right)^{\theta}\left[\frac{1-(1-\delta)/R_{t+1}}{1-\theta}\right]^{1-\theta} \tag{32}$$

To calculate the equilibrium rent when the economy enters the BGP, we need to predict Y_{BGP} and L_{BGP}, the income and land supply in Beijing at $t = T_{BGP}$.

Based on the projected evolution of income, land supply and urban population structure, by the time Beijing market reaches the BGP, per capita income is $Y_{BGP} = 871$ thousand in 2014 RMB, and per capita land supply is $L_{BGP} = 12.83$ square meters. Substituting these numbers and other related values into equation (32), we obtain $r_{BGP} = 4.31$ thousand RMB per square meter. This is the market-clearing annual rent for Beijing $t = T_{BGP}$.

House price at the BGP is calculated as r_{BGP} divided by the price-rent ratio. This gives $P_{BGP} = 153.5$ thousand in 2014 RMB per square meter. Land price at the BGP is 854 thousand RMB per square meter, calculated using equation (15). Thus land price is 5.6 times that of house price at the time the economy enters the BGP.

At the BGP, housing size in Beijing will be $H_{BGP} = 67.36$ square meters per capita based on equation (30). The implied floor-area ratio is $\text{FAR}_{BGP} = H_{BGP}/L_{BGP} = 5.25$. By contrast, in 2014, housing size is about 30 square meters per capita and FAR is below 1.8.

4.3.3 Household Preference Parameters

To pin down the four parameters related to consumer preference, $(\gamma, \mathcal{B}, \beta, \omega)$, we take a moment-matching approach. Specifically, we pick the set of parameters so that the following six moments generated from the model match as closely as possible those from the data: (i) average price-income ratio; (ii) average price-rent ratio; (iii) home ownership rate; (iv) the average age of first-time home buyers; (v) clearing of the rental market in the BGP (equation (27)); (vi) clearing of the housing equity market in the BGP (equation (28)).

To generate model moments, we simulate for each generation 1,000 households with independent idiosyncratic income and medical expense shocks over their life cycle, compute the optimal decisions of each household in the BGP along each path, then calculate the related moments by taking the average values across the 1,000 simulated households.

For each of the moments used in model calibration, Table 2 shows its target value and the fitted value from our calibrated model. The average age of first-time home buyers contains important information about preference for home ownership as well as parameters related to wealth accumulation, such as β and γ. We could not find data on the average age of first-time home buyers in Hong Kong. But the age at first marriage is about 30 according to the Census and Statistics Department of Hong Kong. The age of first-time home purchase is usually older than the age at first marriage, therefore we use 33 as the age of first-time home purchase. As a further reference, in the US, the average age at first marriage is 28 and the age of first-time home purchase is 34 according to the 2009 American Housing Survey.

The home ownership rate target is 0.75 for the Beijing market in the BGP, higher than the rate of 52% in Hong Kong.[21] According to the 2012 wave of the China Household Finance Survey, average home ownership rate in the first-tier cities in China is over 80%. Based on the history of economies that experienced successful economic transition, we do not expect home ownership rate in Beijing to decline significantly in the future.

Table 2: Moments

Moments	Target	Model
price/income	11.88	11.94
price/rent	35.6	35.6
home ownership rate	0.75	0.82
age of first time buyers	33	32
surplus (consumption MKT)	0	0.0
surplus (investment MKT)	0	0.004

Table 3 reports the set of preference parameters with which the model fits best the empirical moments observed in the data. Our model is able to generate a high price-income ratio as well as other moments for Beijing housing market in the BGP, if households are very patient (high β), have a high EIS (low γ) and have a strong bequest motive (high $\mathcal{B}$).

4.3.4 Asset Market Related Parameters

There are four asset market related parameters in our model: risk-free saving rate r_b, and mortgage interest rate r_m, minimum housing size $\underline{s}$, and minimum down payment d. The average annual return on bank deposits is available from the website of People's Bank of China (`http://www.pbc.gov.cn/zhengcehuobisi/125207/125213/125440/125838/125888/index.html`). Between 1990 and 2014, the one-year bank deposit (resp. the 90-day treasury-bill) in China has an average annual return of 1.8% (resp. 1.75%) after inflation adjustment. Chinese banks also offer various "wealth management products" that offer higher returns than bank deposits. Therefore, we set $r_b = 2\%$.

[21]Home ownership rate in Hong Kong is available from Hong Kong Census and Statistics Department, `http://www.censtatd.gov.hk/hkstat/sub/sp150.jsp?tableID=005&ID=0&productType=8`.

Table 3: Parameters

Production parameters		
land share in production	θ	0.72
capital depreciation rate	δ	0.02
scaling parameter in production	Z	1.47
Asset market parameters		
down payment requirement	d	50%
minimum housing size	$\underline{s}$	30
risk-free rate	r_b	2%
mortgage rate	r_m	4%
Preference parameters		
inverse of EIS	γ	1.57
discount factor	β	0.999
housing share in utility	ω	0.30
strength of bequest motive	$\mathcal{B}$	17.29

The mortgage rate in China has ranged from 4.5% to 7% during the past decade. Controlling for inflation, it is about 3-4%. In the baseline model, we set r_m to 4%. In the sensitivity analysis section, we also use $r_m = 3\%$. Minimum housing size is assumed to be 30 square meters per household during the transition period. We also use $\underline{s} = 20$ and $\underline{s} = 40$ in the sensitivity analysis.

We take the minimum down payment requirement d to be 50% in the baseline analysis.[22] In the sensitivity analysis we also consider $d = 30\%$. For home buyers in Beijing, this requirement is typically 30% for the first home of a family, between 50-60% for the second home, and even higher for those purchasing three houses or more.

5 Quantitative Analysis: Results

This section reports the quantitative findings. We first present our main result by comparing the model-implied price and rent in 2014 with the data counterparts, followed by an illustration of an unstable relationship between housing price and fundamentals during the economic transition. Next, we examine the robustness of the key finding by re-calibrating

[22]For home buyers in the first-tier cities, the average down payment ratios is between 45-50% in 2012, as shown in Figure 11 of Fang et al. (2016).

the model under alternative assumptions about land supply, income growth, population structure, mortgage rates, and the reference city. We further extend the model to allow residents in second-tier cities to endogenously make migration decisions. Finally we compare the model-implied outcomes for the pre-2014 period with the historical housing market data we observed.

To ensure proper mapping between the model and the data, we obtain the price between 2005 and 2014 as the weighted average prices of newly-built and existing homes. For each year the weights are the shares of new and existing home transactions in the total value of transactions. The 2014 average house price and rent in Beijing are 28,194 RMB per square meter and 744 RMB per square meter per year respectively, both in terms of 2014 RMB.[23]

5.1 Equilibrium Price, Rent and Other Market Outcomes

Figure 4 reports the trajectories of house price, rent and land price (all in terms of 2014 RMB) that are consistent with the rational decisions of the developer and households. Based on the terminal values (in year 2114) for the Beijing housing market in the BGP that are determined in subsection 4.3.2, we solve for house price and rent trajectories that clear the housing equity market, rental market and land market for each year prior to 2114. Land prices are then calculated from house prices and rents using equation (11).

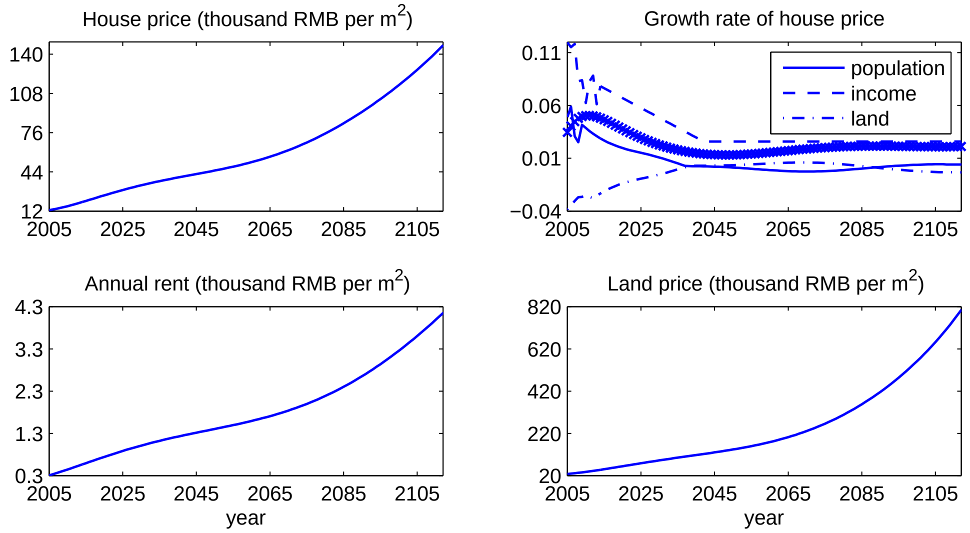
The calibrated baseline model implies an equilibrium house price in 2014 of 19.41 thousand per square meter, which is significantly lower than the 28.194 thousand per square meter in the data, despite the fact that price-rent and price-income ratios of Beijing in the BGP under the model match with the high values of the Hong Kong market. An important factor underlying the relatively low equilibrium price is the more abundant land supply in Beijing. For Hong Kong, given that housing per capita is about 12 square meters and the average floor-area ratio is about 4.5, land use per capita is about $12/4.5 \approx 2.67$ square meters per capita. By contrast, residential land in Beijing is 18.95 square meters per capita. In other words, land use per capita in Beijing is over seven times larger than that in Hong Kong.

The equilibrium annual rent under our baseline model is about 560 RMB per square meter in 2014 – lower than the 744 RMB per square meter in the data.

In the upper-right panel of Figure 4, as indicated by the thick line, the growth of real house price is relatively sluggish between 2040 and 2080, due to an aging population and a period of slow growth of population size after the completion of urbanization. It eventually converges

[23]Details on the data source and the construction of price and rent are reported in Appendix C.

Figure 4: Equilibrium house price, rent and land price

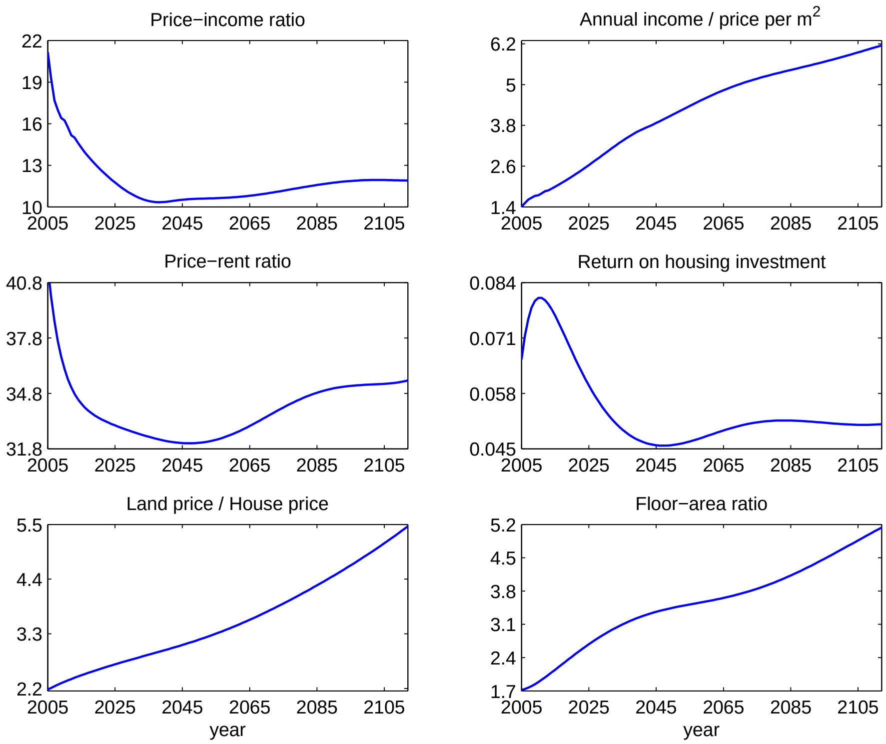


This figure plots the trajectories of house price, rent and land price (in 2014 RMB) under the baseline model. The thick line in the upper-right panel is the growth rate of house price, plotted along with the projected growth rates of population, income and land supply.

to 2.14% – a rate observed in the BGP. The growth rates of population, aggregate income, and aggregate land supply are also plotted. The correlations between these fundamentals and house price are clearly discernible.

Figure 5 reports the trajectories of a number of interesting outcome variables. The upper panels plot two affordability measures: price-income ratio and the ratio of annual income over house price (i.e., how many square meters of housing can be purchased by one year of income). It is clear that the high price-income ratio experienced in the early years of the economic transition period does not necessarily indicate a price bubble. Intuitively, high price-income ratio in the early period is supported by high expected growth rate of income. When income is expected to grow quickly, households' actual ability to pay is much higher than what is captured by the current income. Under the calibrated model, the price-income ratio declines rapidly between 2005 and 2035, then gradually converges to the level in Hong Kong which is about 11.88. Due to the perfect foresight of households, the price-income ratio decreases right from the beginning (year 2005), although the growth rate of income starts to fall only after 2014. The ratio of annual income over house price displays an increasing time trend. Thus, housing affordability improves over time, because the income growth rate is higher than the house price growth rate. In the ***proposition***, we have shown that aggregate income grows more quickly than house price in the BGP. This is also true during

Figure 5: Housing affordability and quantity



This figure plots the evolution of several measures related to housing affordability, housing quantity as well as return on housing investment during the economic transition periods for Beijing market.

the transition period.

The middle panels of Figure 5 plot the price-rent ratio and return on housing investment. Similar to the price-income ratio, the model-implied equilibrium price-rent ratio declines quickly between 2005 and 2045, then rises gradually until it reaches the level in the BGP. The early period of the declining price-rent ratio coincides with a rapid increase in house price and a slower growth in the rent (see Figure 4). Return on housing investment includes both rental return and housing price appreciation. It declines after 2014, which is consistent with the evolution of the exogenous fundamental variables: income growth and the immigration rate both decline over time in our model. The housing return converges to 4.95% in the BGP.[24]

[24] Gyourko et al. (2013) report that the average annual real house price growth between 1950 and 2000 for several "superstar cities" in U.S to be 2.5% or higher. They argue that the high price growth of superstar cities is due to an inelastic supply of land in some unique locations combined with an increasing number of high income households nationally.

The bottom left panel of Figure 5 shows that under our model, the ratio of land price to house price rises steadily. This indicates the increasing importance of land relative to structure in house price, which is consistent with the pattern found in the major U.S. cities (Davis and Heathcote (2007)). Our model also implies that Beijing will witness increasingly higher density, reflected by rising the floor-area ratio as shown in the bottom right of Figure 5.

5.2 Instability of Empirical Relations

Previous studies tend to estimate the relation between house price and economic fundamentals such as income and land supply with linear regressions (Case and Shiller, 2003). This practice works well in developed market. However, in a transition economy where the evolutions of fundamentals are non-stationary, the relation between house price and fundamentals is unstable and time varying. An attempt to estimate the housing price based on its historical relation with the fundamentals could produce misleading results in a transition economy.

Table 4: Linear regression results

Dependent variable:	$log(G_p)$		$log(G_r)$	
	income	land	income	land
2005-2040	0.710	-0.797	0.670	0.321
2041-2080	1.161	-3.020	0.761	-1.135
2081-2114	0.807	0.140	0.856	-0.916

This table reports the estimated coefficients when we regress log price growth (or log rent growth) on log income growth and log land supply growth using the equilibrium path of price or rent and the exogenous fundamentals, for various subperiods during the economic transition.

To illustrate this point under our calibrated model, we run two regressions using simulated data to contrast the relation between house price and economic fundamentals in the BGP and during the transition periods. First, from points 5-6 in the ***Proposition***, we have the following in the BGP:

$$log(G_p) = \theta log(G_Y) - \theta log(G_L) \tag{33}$$

$$log(G_r) = \theta log(G_Y) - \theta log(G_L) \tag{34}$$

For the Beijing market, when we regress log-price growth on $log(G_Y)$ and $-log(G_L)$ restricting the coefficients to be identical using simulated data in the BGP, we recover a coefficient of 0.72, just as the model predicted (recall that we use a parameter value $\theta = 0.72$ in the calibrated model). The same holds when we use log-rent growth as the dependent variable.

Second, we run another regression using the equilibrium path of price or rent and exogenous fundamentals during various subperiods of the economic transition, but without requiring the coefficients before $log(G_Y)$ and $-log(G_L)$ to be the same. Table 4 reports the results. The difference in the estimated coefficients across various periods is evident. Theoretically, for a transition economy, the relation between house price (or rent) and the fundamentals such as income, land supply and age distribution of population can be non-linear and time varying because of the non-stationarity of the variables. Linear regressions assuming constant parameters would fail to capture these complex dynamic dependences.

5.3 Sensitivity Analyses

The baseline model implies an equilibrium house price in 2014 of 19,410 RMB per square meter that is significantly lower than the 28,194 RMB observed in the data. In this section, we check the robustness of this result under alternative model specifications.

5.3.1 Alternative Projection of Land Supply

In the baseline model, we have assumed that aggregate land supply grows at a rate of 0.5% after 2014. To gauge the sensitivity of the quantitative results with respect to land supply, we examine two alternative assumptions: the growth rate of land supply is either 0% or 1% after 2014. The corresponding land supply per capita is shown in the right panel of Figure 2 (dashed and dotted lines respectively). The market clearing paths of prices and rents are recomputed, and the results are summarized in the rows labeled (1) and (2) in Table 5.

With more abundant land supply, house price and rent as well as their growth rates are uniformly lower compared with the baseline case. For example, house price increase between 2014 and 2114 reduces from about eightfold when land supply grows at 0.5% per year to less than fivefold when land supply grows at 1% per year.

In the case of zero land supply growth, both house price and rent when the economy enters the BGP are about 35% higher than the baseline case of 0.5% land supply growth. House price in 2014 under the assumption of zero growth in land supply is 22.33 thousand RMB per square meter. This is 15% higher than the baseline case, but still significantly lower than the data. Thus we conclude that more stringent land supply does not help much to narrow the gap between the model-implied house price and that in the data.

5.3.2 Alternative Projection of Income Growth

The baseline scenario assumes that the growth rate of aggregate income declines linearly from 7% in 2015 to 3% in 2045. Table 5 rows (3)-(4) report the equilibrium house prices and rents assuming a shorter (20 years) or a longer (40 years) period of high income growth.

Take the case where the income growth declines to 3% over the next 20 years as an example. Compared to the baseline scenario, the future income path is uniformly lower under this alternative case. This has two effects. First, households now have a stronger incentive to save for the future due to the substitution effect, leading to a higher housing investment demand but weaker housing consumption demand. Second, there would be less housing demand – for both consumption and investment purposes (see the right panel of Figure 2), due to the income effect. The impact on rent is unambiguously negative because both effects work in the same direction for housing consumption demand. But the two effects have opposing implications for housing investment demand and thus the house price. The net impact on house price depends on the relative strength of the two effects.

Qualitatively, we find that under the alternative case of a shorter period of high income growth, house rent and price in the BGP are both significantly lower than the baseline case, suggesting that the income effect dominates the substitution effect. Intuitively, the income effect remains strong throughout the transition periods because the difference in the income levels between the two cases continues to widen as shown in the right panel of Figure 2. In contrast, the substitution effect gradually phases out over time (there is no difference in income growth rate between the two scenarios after 30 years). Similarly, the income effect also dominates the substitution effect during the economic transition periods. Thus, the equilibrium house price and rent in 2014 are lower than the baseline results. Conversely, when the high income growth lasts for a longer period, the implied house price and rent are higher throughout the transition period.

Quantitatively, the equilibrium house price and rent under our model are not sensitive to alternative income growth assumptions. For example, Table 5 rows (3)-(4) show that the model-implied house price in 2014 is 18.55 (resp. 20.06) thousand RMB per square meter corresponding to a shorter (20 years) or a longer (40 years) period of high income growth. These price levels are within 5% of the Beijing house price under our baseline model.

5.3.3 Alternative Projection of Migration

In the baseline model we assume the rate of migration to Beijing decreases linearly from 2.88% in 2014 to zero after 30 years. To understand how sensitive our results are to this assumption, we recompute the equilibrium using a 40-year period of decrease, instead of a 30-year period. Results are reported in row (5) of Table 5. Extending the period of migration generates more demand for both housing consumption and housing investment, hence it yields higher house price and rent compared with the baseline results, especially in the long-run. For example, with 10 additional years of migration, the house price in 2014 (resp. 2114) is 5% (resp. 32%) higher than that under the baseline model. The magnitude of the effect of longer duration of migration is much more significant in 2114, although in both models migration stops after 2054. This is because the population base is larger in 2054 which, when combined with a growing trend of population, leads to a much larger population in the alternative model in 2114.[25]

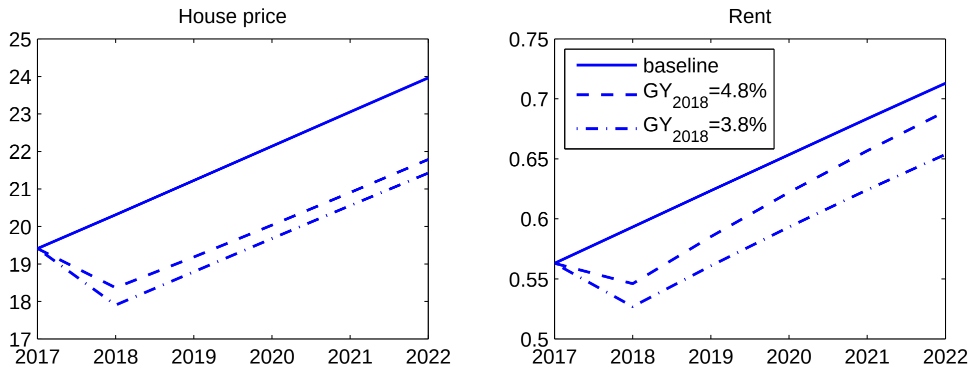
5.3.4 Unexpected Change in Income Growth

In this subsection, we conduct a robustness exercise regarding the quantitative implications of an unexpected change of income growth. A sudden decline in the growth of disposable household income is one of the major risks facing the current Chinese housing market (Fang et al. (2016)). To capture this possibility, we assume that households make optimal choices based on expectation of future income growth as assumed in the baseline model, until in the beginning of 2018, a sudden drop in income growth is realized, and then they adjust expectation of future income growth downwards and re-optimize accordingly.

Figure 6 plots the paths of price and rent when income growth unexpectedly drops from 5.8% in 2018 (under the baseline scenario) to either 4.8% or 3.8% and subsequently declines linearly to 3% by 2044. In response to an unexpected negative income shock, both house price and rent drops immediately, then grows steadily again over time, albeit from a lower base. The reason is that a sudden decline of income growth brings strong and immediate income effect. The decline of house price when income growth drops from 5.8% to 4.8% is sharp, but the further drop of income growth to 3.8% causes only mild decline in house price. In the case that income growth in 2018 drops to $GY_{2018} = 3.8\%$, house price and rent decline by 11.9% and 11.2% respectively. By the time the economy enters the BGP, house price and rent are 133.45 and 3.76 thousand RMB. Compared with the baseline results, the case of $GY_{2018} = 3.8\%$ lowers house price and rent in the BGP by 13.1% and 12.8% respectively.

[25] Note that our model does not allow indefinitely continuing migration at the BGP. This is consistent with the previous literature (e.g., Garriga et al. (2016)). Despite this assumption, the overall population continues to grow at a rate of 0.4% at the BGP, which is close to the growth rate in Taiwan after 2000 and higher than the comparable growth in New York city after 2000.

Figure 6: House price and rent in response to an unexpected drop in income growth



This figure plots the paths of price and rent when the income growth rate drops suddenly in 2018.

5.3.5 Alternative Initial Wealth

We now consider raising higher average household initial financial wealth (in the year 2005) from 73.65 thousand RMB in the baseline scenario to 200 thousand RMB. Results are reported in row (6) of Table 5. With this higher initial wealth, the implied house price is 20 thousand RMB per square meter in 2014, a rise of 3% relative to the baseline. Correspondingly, the rent increases by 5.35%. Thus, the effect of a large increase (almost threefold) in the initial household wealth on equilibrium house price and rent is very small.

In an earlier version of the model where there is no risk-free asset and housing is the only investment vehicle, we find a much stronger impact of initial wealth on house price, but again a marginal impact on rent. Corresponding to the same increase of average initial wealth rom 73.65 thousand to 200 thousand RMB, house price and rent in 2014 increase by about 24% and 5.4% respectively. Intuitively, households use the extra wealth to smooth out their consumption over time, hence the marginal effect on current rent. But in the case where housing is the only investment asset available, a higher initial wealth implies an immediate jump in housing investment demand, hence the more significant rise in house price.

5.3.6 Upper Bound on Floor-area Ratio

We have shown that, in the absence of building restriction, the floor-area ratio (FAR) in Beijing should be between 5 and 6 when the economy enters the BGP. Afterwards, it will grow at the constant annual rate of $(G_Y/G_L)^{1-\theta} - 1$ which is about 0.82% according to our calibration. Here we ask what happens if the government imposes an upper-limit on FAR, denoted $\overline{FAR}$, so that urban density is restricted. This question has real-world relevance. In March 2008, the Beijing City Planning Committee and Beijing City Land Resources Bureau jointly issue a policy called "*Beijing City Construction Land Saving Standards*", stipulating that the FAR of residential area should not exceed 2.8.

In row (7) of Table 5, we assume $\overline{FAR} = 3.0$ and re-compute the equilibrium house prices and rents.[26] By potentially limiting house supply, the policy of an upper bound on FAR lead to an increase in house price and rent by 10.4% and 8.9% respectively in 2014, and for house price by 89% and for rent by 84% in 2114 relative to the baseline scenario.

5.3.7 Idiosyncratic Uncertainty

In this subsection we check how the main results change depending on the amount of idiosyncratic risk faced by the households, as represented by the standard deviations of income and medical expense shocks. This exercise not only helps us quantify the importance of precautionary savings, but also provides insights about potential benefits from policies such as social insurance and medical insurance programs.

The presence of medical expense shocks and income shocks give the households a precautionary motive to save in our model. The precautionary demand for housing equity increases with the idiosyncratic risks and housing consumption decreases with the idiosyncratic risks. Thus, other things equal, a larger idiosyncratic risk would lead to higher equilibrium house price and lower rent in our model.

If the standard deviation of income shocks is cut by 50%, then we observe a lower house price but a higher rent, both in 2014 and 2114, as shown in row (8) of Table 5. This is caused by a reduced precautionary demand for housing asset and the stronger consumption demand due to the lower income risk. Quantitatively, house price is lowered by 3% and rent is raised by 0.6% in 2014 compared with the baseline model. By the time the economy converges to the BGP, house price is lowered by 6% and rent is raised by 1.9%.

Row (9) of Table 5 reports the results when we cut the standard deviation of medical expense shocks by 50%. The effect is qualitatively the same as that of lower income risk, but the magnitude is smaller. In an unreported sensitivity check, we completely remove the idiosyncratic shocks. This causes the current house price to drop by 9.4% and rent to rise by 15.8%. Thus the precautionary savings motive created by idiosyncratic income and medical expense shocks plays a sizable role in generating high house price and price-income ratio in

[26] Technically, whenever the implied FAR exceeds $\overline{FAR}$, we set housing supply to $\overline{FAR} \times L^*$, where L^* is the exogenous land supply.

the Beijing market.

5.3.8 Alternative Housing Market Assumptions

The baseline model uses a mortgage borrowing rate r_m of 4% and a minimum down payment d of 50%. The minimum housing size $\underline{s}$ during the transition periods is 30 square meters. To check the sensitivity of our results, we change these parameters one at a time, seting $r_m = 3\%$, or $d = 30\%$, or $\underline{s} = 40$ respectively. Results are reported in rows (9)-(11) of Table 5.

Lowering the mortgage rate or down payment requirement makes housing more affordable, leading to an increase in housing demand and house price. The effect is quite significant. For example, Row (10) of Table 5 shows that by lowering the down payment from 50% to 30%, house price and rent in 2014 increase by about 26.8% and 12.5% respectively. Similarly, row (11) shows that house price increases by about 13% when the mortgage rate is cut from 4% to 3%.[27] Nevertheless, even with easier access to the housing market, the equilibrium house price in 2014 is still substantially lower than the data.

Increasing the minimum house size serves to depress the equilibrium house price but increase the rent, because now more households have to rent which reduces the housing demand. But quantitatively, changing the minimum housing size has a rather small effect on the equilibrium house price. When the minimum house size is increased from 30 to 40 square meters, price drops by 2.6% from 19.41 thousand RMB to 18.89 thousand RMB per square meter in 2014, as shown in row (12) of Table 5. In an unreported robustness check, we find that lowering the minimum house size to 20 square meters barely changes the current house price compared to the baseline result. This is expected because the average housing size for Beijing residents is already 19.5 square meters in 2005, the initial year of our analysis, so a minimum housing size of 20 square meters usually is not a binding constraint. The high down payment requirement represents a larger barrier to entry in the housing market.

[27] In an unreported robustness exercise, we examine the impact of an increase in the risk-free interest rate or mortgage rate. When the risk-free rate rises from 2% to 2.5%, the equilibrium house price decreases by 1.54%, and the rent increases by 0.65%. The change of risk-free rate has both an income effect and a substitution effect. These two effects appear to largely cancel each other out. When the mortgage rate rises from 4% to 5%, the equilibrium house price decreases by 13.3%, and the rent decreases by 8.3%. The higher mortgage rate effectively reduces the leveraged return on housing investment, leading to both a negative income effect and a negative substitution effect. Thus the impacts on house price and rent are large.

Table 5: Price and rent under alternative assumptions (in thousands of 2014 RMB)

		Price		Rent	
		2014	2114	2014	2114
	Data	28.19	n.a.	0.74	n.a.
	Baseline	19.41	153.54	0.56	4.31
(1)	Growth of land supply = 1%	18.10	86.40	0.57	2.72
(2)	Growth of land supply = 0%	22.33	207.21	0.62	5.90
(3)	Income stabilizes in 20 years	18.55	128.87	0.54	3.64
(4)	Income stabilizes in 40 years	20.06	183.81	0.60	5.32
(5)	Migration stops in 40 years	20.39	203.24	0.59	5.56
(6)	Initial financial wealth = 2 M	20.01	161.07	0.59	4.42
(7)	FAR ≤ 3	21.43	289.87	0.61	7.94
(8)	Half idiosyncratic income risk	18.83	144.32	0.57	4.39
(9)	Half medical expense risk	19.37	151.79	0.57	4.31
(10)	Minimum down payment = 30%	24.62	179.96	0.63	4.66
(11)	Mortgage rate = 3%	21.96	176.41	0.62	4.65
(12)	Minimum housing size = 40 m^2	18.89	153.36	0.57	4.36
(13)	Washington, D.C. (full recalibration)	23.69	105.34	0.81	3.39
(14)	Washington, D.C. (partial recalibration)	14.87	122.81	0.53	3.95

This table reports the equilibrium price and rent for the Beijing housing market in 2014 and 2114 under alternative model assumptions.

5.3.9 Alternative Reference City

Thus far Hong Kong has been used as the reference city to pin down parameters related to household preferences and housing production. This might raise a couple of legitimate concerns. First, Hong Kong's housing market could be affected by the current state of the Beijing market; and second, land area is confined in Hong Kong but open and expandable in Beijing. To address these concerns, we recompute the equilibrium using Washington, D.C. as an alternative reference city. Unlike Hong Kong, housing market in Washington, D.C. is minimally tied to Beijing. In addition, Washington, D.C. is similar to Beijing in that land is open and amenable to new construction.

In the recalibration, we start with four household preference parameters $(\gamma, \mathcal{B}, \beta, \omega)$. These parameters are pinned down by matching the moments as shown in Table 2, with the price-income ratio and price-rent ratio of 31.1 and 9.08 respectively, calculated as the post-2010 average ratios observed in Washington, D.C. The resulting parameters are γ=2.09, $\mathcal{B}$=16.32, β=0.998, and ω=0.25. Compared with the baseline calibration, a larger γ indicates a smaller EIS while a smaller ω indicates weaker consumption demand for housing.

For housing production parameters, we first pin down the land share parameter θ using equation (29) based on the growth rates of house price and land price in Washington DC.[28] This gives $\theta = 0.60$, which is smaller than $\theta = 0.72$ in the baseline calibration. The lower θ arises from the relatively abundant land supply in Washington, DC which yields a lower growth rate of house price relative to land price compared to Hong Kong.

To pin down the production efficiency parameter Z, we use equation (9) which implies $Z = FAR^{\theta} / \left[\frac{(1-\theta)r}{1-(1-\delta)/R}\right]^{1-\theta}$, where FAR, the floor-area ratio in the BGP, is taken to be 3.[29] Using $r = 16.53$ USD per square foot and $R = G_p + (r/p) = 1.05$, we obtain $Z = 0.90$ which is substantially smaller than $Z = 1.47$ in the baseline calibration. The smaller value of Z reflects the lower rise buildings featured in the reference city compared to Hong Kong.

Finally, we infer the growth of land supply (G_L) from the sixth point in the ***Proposition***, $G_p = (G_Y/G_L)^{\theta}$. For Washington, D.C., we obtain $G_Y = 1.056$ and $G_p = 1.020$, which yields $G_L = exp(log(G_Y) - 1/\theta log(G_p)) = 1.02$. This is about the average land supply

[28] Using the 1987-2015 Case-Shiller house price index, the real growth rate of house price is 1.94%. The growth of land price between 1987-2015 is taken from the Lincoln Institute (`http://www.lincolninst.edu/subcenters/land-values/metro-area-land-prices.asp.`).

[29] This number is lower than the present FAR in Hong Kong, but higher than the present FAR in Washington, D.C. Housing in D.C. is mainly stand-alone single-family houses while in Beijing, it is predominately condominiums. FAR=3 in the BGP is a more realistic assumption for Beijing because the FAR of condominiums built in recent years in Beijing already exceeds 2.5.

growth of seven U.S. cities featuring rich land supply: Chicago, Cincinnati, LA, Minneapolis, Philadelphia, Pittsburgh and Tacoma.[30] Note that with 2% of land growth instead of 0.5% as in the baseline specification, Beijing has acquired much more land expansion opportunities when its future path is based on Washington, D.C.

Row (13) of Table 5 reports that under the above full recalibration with Washington, DC as the reference city, Beijing house price in 2014 is 23.69 thousand RMB. Compared to the case of Hong Kong, using Washington, DC to resemble the future of Beijing lowers housing demand (reflected by smaller ω), putting a downward pressure on equilibrium price. However, the supply side effects on house price are mixed. On the one hand, higher land growth (reflected by larger G_L) increases land supply. On the other hand, lower building density and height (reflected by lower θ and Z) effectively reduce the efficiency of land use.[31] It turns out that the production efficiency effect that tightens housing supply dominates the demand effect and the land growth effect, yielding a higher equilibrium price than in the baseline specification. Nevertheless, the model-implied price in 2014 is still 16% below the market price, suggesting that the substantial gap remains between model predicted and observed market prices.

One concern with the above full recalibration is that low urban density and building height in Washington, DC do not present a realistic analogue for the urban configuration of Beijing. To address this, we further present a partial recalibration where housing production parameters remain the same as in the baseline specification but household preference parameters are recalibrated to match the Washington, DC data. The resulting building density and floor-area ratio are closer to what we observe in Beijing. By restricting the supply side as in the baseline calibration, we should expect that the demand side effect will dominate and hence the equilibrium price should be lower compared to the baseline specification. Row (14) of Table 5 shows that the model-implied price is 14.87 thousand RMB in 2014, which is about half of the market price.

5.4 Reconciling the Model-data Discrepancy

We have shown in the baseline model and in various sensitivity analyses that the model-implied equilibrium house price and rent are substantially lower than the actual data. The

[30]See `http://www.lincolninst.edu/subcenters/atlas-urban-expansion/global-sample-cities.aspx`.

[31]With Washington DC as a reference city, FAR is 3 when Beijing enters the BGP as opposed to 5.2 as in the baseline case; housing size per capita is 36.7 square meters in the BGP as opposed to 65 square meters as in the baseline calibration.

discrepancy could come from a number of channels, including: (i) measurement errors in the government data on housing fundamentals (e.g. household income) we use; (ii) strong model assumptions (e.g. exogenous population inflow); (iii) certain factors or market features that our model abstracts from (e.g. aggregate uncertainty, demand shocks, search frictions in the housing market); (iv) housing bubbles. While channels (iii) and (iv) are beyond the scope of our paper, in this subsection we experiment with channels (i) and (ii). Specifically, we study the impact of understatement of household income under the baseline model. We also examine an extended model that allows sorting of households into Beijing as an outcome of migration choices.

5.4.1 Understatement of Income

The average income of Beijing residents as reported by the National Bureau of Statistics (NBS) can be understated for several reasons. First of all, a large number of rich households outside of Beijing purchase houses in Beijing for a variety of reasons including as an investment, and for better access to facilities, services and conveniences. Some of these home buyers live in Beijing for only short periods of time hence they are not included in the NBS survey. Second, government officials sometimes report extremely mediocre income although their actual income is considerably higher than the average.[32] Deng et al. (2016) estimate that the amount of unofficial income for an average government official is 60% of his or her official permanent income. Indeed, they argue that this may be an underestimation and the actual unofficial income could be even bigger.

Row (2) of Table 7 reports the results of our model when the level of income in Beijing is raised by 60% while the income growth rate remains the same as the baseline model. Given the uniformly higher income, both house price and rent increase unequivocally. The model-implied house price in 2014 is 27,040 RMB per square meter, which is close to the 28,194 RMB market price in the data. The equilibrium annual rent is 800 RMB per square meter, compared to 740 RMB observed in the data. In other words, the level of price and rent in 2014 are roughly consistent with the fundamentals if the average income of Beijing residents is about 60% higher than reported in the government statistics. Although an understatement of income of 60% is found to be plausible for government officials, we do not think such substantial income understatement can represent the overall population, nor can it be sustainable in the long run. Thus one cannot rely on the income understatement to completely close the gap between the model-implied price and the market price.

[32] Recent years have seen increasing news reports about officials owning dozens of condominiums in Beijing.

5.4.2 Endogenous Migration

In the baseline model, we have taken the inflow of migrants to Beijing as exogenously given. In reality, the only people who would move are those who have found that the benefits of migration outweigh it costs. In this subsection, we extend the baseline model to study the endogenous migration of households. Note that our goal here is not to formally investigate the driving force behind the labor mobility, rather to examine to what extent the main finding would change when the non-randomness of the population inflow is taken into account. Formally modelling the labor reallocation dynamics, particularly the urbanization process, could be revealing but is beyond the scope of this paper. In this regard, recent work by Garriga et al. (2016) has provided an elegant dynamics general equilibrium model that links urbanization to the ongoing structural transformation in China.

In each period, a potential migrant decides whether to move by comparing the life-time utility she receives if she stays in her hometown with the life-time utility she receives if she moves to Beijing. As in the baseline model, the utility in each scenario is computed based on her optimal tenure, consumption and investment decisions. Our characterization of migration benefits and costs is consistent with the literature. A migrant moving to Beijing faces higher housing costs than if she stays in the home market. On the other hand, she receives utility gains that capture improved amenities ((Henderson and Becker, 2000)), better income prospects that capture the productivity difference (Garriga et al. (2016)), and higher return on housing that she owns in Beijing.

Specifically, the utility function becomes $u(\phi c, \phi h)$ where ϕ measures, in the form of consumption equivalence, the advantage of migrating to Beijing in the following way:

$$\phi \begin{cases} = \phi^{owner}, & \text{if migrate to Beijing and own;} \\ = \phi^{renter}, & \text{if migrate to Beijing and rent;} \\ = 1, & \text{otherwise.} \end{cases}$$

The utility gain parameter ϕ differs across renters and homeowners. Turning to income, we assume that if a household i migrates to Beijing, her subsequent income is given by $y(i,a,t) = \tilde{y}(i,a,t) \times \overline{y}(a,t)$, where the deterministic component $\overline{y}(a,t)$ is the average income of Beijing residents of the same age, but the individual-specific stochastic component of income $\tilde{y}(i,a,t)$ is not affected by the migration decision.

When taking the model to the data, we restrict our attention to potential migrants who are from the so-called second-tier cities in China.[33] The total population in these cities is

[33]Thus we omit the rural-to-urban migration. If we replace some of the migrants from second-tier cities

about 10 times the Beijing population. Realistically, only a fraction of people outside Beijing consider moving, we therefore randomly draw one-third of the population of the second-tier cities as the pool of potential migrants to Beijing.[34]

To obtain information about migrants' home markets, we consider four representative second-tier cities: Wuhan, Chengdu, Dalian and Xi'an. As can been seen from the upper panel of Table 6, their average house price and annual rent in 2014 are around 9,000 RMB and 300 RMB per square meter respectively, both less than half of the corresponding values for Beijing. The average disposable income per capita in these cities is about 3/4 of the Beijing income level.

Table 6: A comparison of out- and in-migrating cities

	Second-tier City	Beijing
Data		
House Price (per m^2)	9,000	30,000
Rent (per m^2 per year)	300	700
Income	32,933	43,910
Initial Housing Asset	30 m^2	30 m^2
Initial Financial Asset	3.68 × income	3.68 × income
Assumptions		
Growth of Price (BGP)	1.62%	2.14%
Growth of Rent (BGP)	1.62%	2.14%
Price/Rent (BGP)	30	35.6
Growth of Rent (transition)	exogenous	endogenous
Growth of Price (transition)	exogenous	endogenous
Growth of Land supply	0.077%	0.05%

Further, we make the following assumptions about the second-tier cities in the BGP. First, the price-rent ratio is 30 – this is slightly lower than that of Beijing. Second, return on housing investment is 4.95% – this about the same as in Beijing. Together, these two

with those from the rural areas, it would lead not only to reduced home ownership among migrants, but also reduced home purchases conditional on owning. This would bring down the equilibrium price and magnify the gap between the predicted and actual prices even further. Thus, the conclusion of the main analysis remains robust.

[34]Our results are not sensitive to this choice as long as we allow enough households outside of Beijing to consider moving so that the migration rate target can be matched.

Table 7: Prices and rents (in thousands of 2014 RMB)

		Price		Rent	
		2014	2114	2014	2114
	Data	28.19	n.a.	0.74	n.a.
(1)	Baseline	19.41	153.54	0.56	4.31
(2)	Income Raised by 60%	27.04	211.34	0.80	6.03
(3)	Endogenous Migration	21.84	152.17	0.51	4.24

This table compares the equilibrium price and rent for Beijing housing market under alternative model specifications to the data.

assumptions imply that the growth rate of house price in the BGP is 1.62% for second-tier cities – lower than the 2.14% rate for Beijing. Finally, the income growth rate of the second-tier cities in the BGP is assumed to be the same as that of Beijing.

It follows from point 6 of the ***Proposition*** that the growth factors of land (G_L^0) and house price (G_p^0) for second-tier cities in the BGP are related by

$$G_L^0 = G_L \left(\frac{G_p}{G_p^0} \right)^{\frac{1}{\theta}}, \tag{35}$$

where G_L and G_p denote the growth factors of land and house price for Beijing. Substituting $G_L = 0.05\%$, $G_p = 2.14\%$ for Beijing and $G_p^0 = 1.62\%$, we obtain a land supply growth $G_L^0 = 0.077\%$ for second-tier cities, which is higher than the land growth rate of 0.05% in Beijing. Under these assumptions (as summarized in the lower panel of Table 6), we compute the endogenous migration decisions of potential migrants as well as the equilibrium paths of house prices and rents for Beijing and second-tier cities during the economic transition period.

The extended model has two additional parameters ϕ^{owner} and ϕ^{renter} that measure the utility gain of migrating to Beijing as a homeowner or a renter, respectively. We calibrate these parameters by targeting the following two moments based on the data: (i) the migration rate (number of migrants divided by Beijing population) equals 2.88% in 2014; (ii) the ratio of average income of migrants to those with Beijing Hukou is 0.86 (based on the 2005 mini-census data). The calibrated model parameters are $\phi^{owner} = 1.28$ and $\phi^{renter} = 1.23$. The difference captures benefits that are only available to homeowners in Beijing, such as child education benefits.

In the equilibrium, there are both rich and poor households that optimally choose to move to Beijing, with the latter group starting out as renters, just like their local resident counterparts. The last row of Table 7 reports price and rent from the endogenous migration model. As of 2014, house price in Beijing is 21,840 RMB per square meter, which is 12.5% higher than the baseline model but 23% lower than the market price. The annual rent in 2014 is 510 RMB per square meter, lower than that in the baseline model (560 RMB) as well as in the data (700 RMB). Overall the equilibrium price and rent are still significantly lower than what we observed in the data. Thus our main finding about the gap between the model-implied and actual home price remains robust.

The lower equilibrium rent relative to the baseline model is expected. In the simulated model with endogenous migration, the average income of migrants who rent in Beijing is 83% of that of local renters. This leads to a lower housing consumption demand and hence a lower rent in the model with endogenous migration, compared to the baseline model where migrants and locals face the same income profile.

On the other hand, in the extended model, the higher equilibrium price relative to the baseline model reflects migrants' stronger incentives to save than Beijing residents, because moving to Beijing gives migrants higher utility (as captured by $\phi > 1$). Compared to an average local resident, an average migrant has accumulated more wealth by the time she purchases a house in Beijing.[35] The positive effect of migrating homeowners' larger asset holdings on housing investment demand dominates the negative effect due to their lower average income. As a result, the equilibrium house price in the endogenous migration model is higher than that in the baseline model.

To further assess the quantitative performance of the extended model, we compare the model predicted income ratio between migrants and locals as well as homeownership difference between them with the data counterparts. The model implies that the average income of migrants in 2014 is 92% that of incumbent Beijing residents, which is slightly higher than the 86% income ratio as observed in the data. In the simulated model with endogenous migration, the homeownership rate is 74.8% for migrants and 85.6% for Beijing local residents in 2014. In the data, average home ownership rate is about 42% for migrants and about 82% for local Beijing residents.[36] Thus overall the model does a reasonable job, particularly for

[35]In the simulated model, migrants who purchase house in Beijing have an average housing size that is 15% larger than that of local homeowners in Beijing, although the average income of the former is 3% lower than the latter.

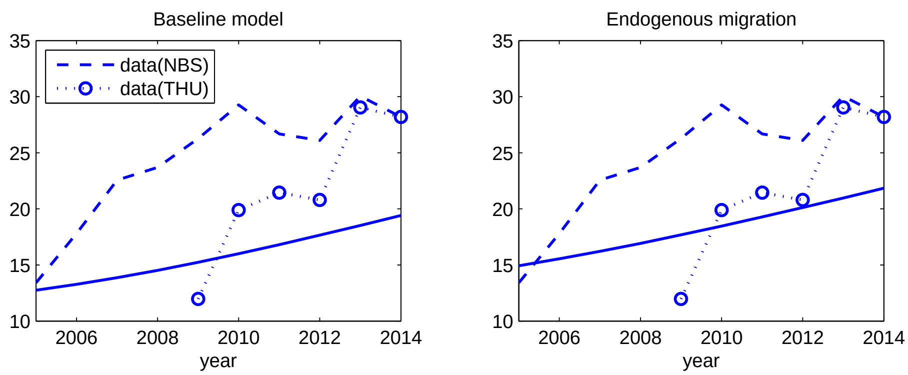
[36]Home ownership rate for Beijing is proxied by the average rate in the first-tier cities from the 2012 wave of China Household Finance Survey. The home ownership rate for migrants is from `http://www.360doc.com/content/10/0711/07/72265_38196048.shtml`. In addition, Table 2 in Wu et al. (2012) shows that

local residents. The over-prediction of home ownership and income for migrants is expected given that the endogenous migration model excludes rural-to-urban migration.

5.5 Equilibrium Outcomes Compared with the Historical Data

We now compare the model-implied house prices, price-income ratio and price-rent ratio with what we observed in the pre-2014 period. Two series of house prices are constructed, referred to as the NBS series and the THU series respectively. The former is quality-unadjusted while the latter is quality-adjusted. Both series are calculated as the weighted averages of newly-built and existing homes. Details on the construction of historical house prices as well as the price-income and price-rent ratios are provided in Appendix C.

Figure 7: House Price 2005-2014



This figure compares equilibrium house prices from the model with those from the data. The "data(NBS)" series is quality-unadjusted and the "data(THU)" series is quality-adjusted.

Figure 7 plots the equilibrium house price from the baseline model and the endogenous migration model from 2005 to 2014. In each case, the model-implied price is in line with the data in 2005. However, the house price growth rate between 2005 and 2014 is lower than that in the data. Consequently the model-implied price in 2014 is significantly below the price in the data. One way to measure the fit of the model is the normalized mean squared error (NMSE) proposed by Garriga, Tang and Wang (2016). NMSE is defined as $\sum_t (x_t^M - x_t^D)^2 / \sum_t (x_t^D)^2$, where x_t^M and x_t^D are the prices from the model and from the data respectively. For the NBS series, the NMSE is 0.13 from the baseline model and 0.08 from the endogenous migration model. For the THU series, the NMSE is 0.08 from the baseline model and 0.05 from the endogenous migration model. Thus the overall fit of house prices

about one third of the housing units in China were purchased by migrants in 2009.

is good.

We also compare the price-income and price-rent ratios from 2005 to 2014 implied by the model to those based on the data using the weighted averages of prices and rents for existing and newly-built homes, as detailed in Appendix C. Figure 8 shows that the model does not capture the rising price-income ratio and price-rent ratio in the data prior to 2010. One possible reason for this wedge is that households in our model perfectly foresee that high income growth will slow down in the future, which generates a declining price-income ratio. In reality, evidence suggests that during 2005-2010, Chinese households likely adopted unrealistically optimistic expectations of future income growth.[37] High income growth expectations, combined with the access to the mortgage market, make housing more affordable, causing a rising price-income ratio over time as observed in the data. The unrealistic expectations about future income growth also lead to a rising price-rent ratio, since the high growth rate of income is associated with high capital gain in the housing market, which lowers the required rental return.

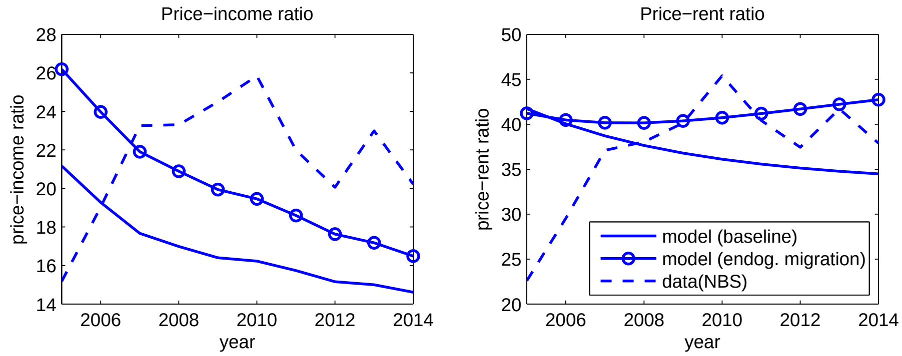
In the subperiod of 2010-2014, both models perform better than in the first subperiod: the baseline model captures the declining patterns in the price-income and price-rent ratios; the endogenous migration model captures the declining price-income ratio. It is possible that the global financial crisis made households more aware of market conditions or the potential for mean reversion in the income growth rate. Therefore, expectations may have conformed more closely with what our model assumes.[38]

Finally, we compare the age distribution of home ownership rate and the life-cycle profile of housing assets in the data to those implied by the model. The left panel of Figure 9 plots the model-generated age distribution of the home ownership rate in 2010 and that calculated from the 2010 census data. In the data, the home ownership rate starts at about 31% at 21, rises gradually to about 86% at age 75 and then falls slightly afterwards. The average home ownership rate is 66.5%. In the model, home ownership rate rises monotonically from about zero for those in their early twenties to almost 100% by age 65. The average home ownership rate under the model is 81.8%, higher than the data. This gap occurs because housing investment has a high return and no risk under our model; as a result, households

[37]As noted in Fang et al. (2016), "such high income growth expectations might have resulted from extrapolative behavior as emphasized by Barberis et al. (1998) and Shiller (2000), or from contagious social dynamics between households as modeled by Burnside et al. (2016)."

[38]Our model ignores cyclical movements and hence does not capture the temporary rise in the price-income and price-rent ratios between 2012 and 2013 which may could be due to changes in down payment and mortgage rate caused by government interventions.

Figure 8: Price-income and Price-rent Ratios



Note: This figure shows the price-income ratio and price-rent ratio under the baseline model and the model with endogenous migration, along with the data counterpart. House price series is based on the weighted average of new and existing house prices as reported by the NBS. Rent series is from the rental index for Beijing as reported by the NBS based on the rents of both new and existing homes.

become homeowners as soon as they can afford the minimum down payment.

The right panel of Figure 9 plots the life-cycle profile of housing assets from the model in 2012 along with the national level age distribution of housing size (in square meters) provided by the 2012 China Household Finance Survey. Although we do not target to fit this age distribution in 2012, the model-generated age distribution of housing asset is highly correlated with the data. The magnitude of housing asset under our calibrated model is usually somewhat larger than that in the data, except for both very young and old households.[39]

6 Conclusion

This paper presents a dynamic rational-expectations general equilibrium model that links house price to economic fundamentals including income growth, land supply, population structure and migration. Our model is general enough to deal with non-stationary fundamentals in emerging markets. It can be used to generate rich predictions about the dynamics of house price and housing affordability from variations in both the supply and demand sides of the housing market. We apply the model to the Beijing market, and examine to what extent current house prices are consistent with rapidly changing economic fundamentals. We

[39]Part of the deviation is due to the data imperfection: we do not have access to data on housing assets of Beijing residents. Instead, the data we used is at the national level.

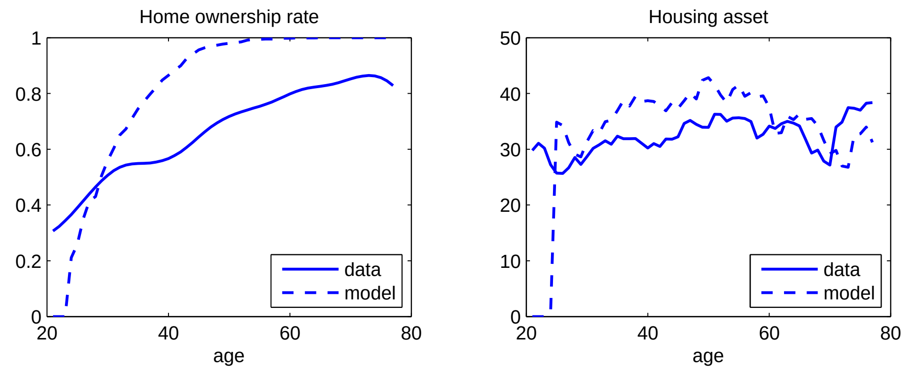


Figure 9: Home Ownership Rate and Housing Asset: Model versus Data

Note: This figure plots the age distribution of the home ownership rate and housing asset (in square meters per capita) in the data and implied from the model.

also develop an efficient numerical method to compute the paths of equilibrium house price and rent during the economic transition.

Under reasonable parameterizations, the model-implied equilibrium house price for Beijing is significantly lower than the market price. This gap between model and market home price is robust to alternative assumptions about land supply, income growth, population structure, mortgage rate, down payment requirement, and choice of the reference city. The discrepancy between the predicted and actual prices could come from a number of channels, such as income underreporting, sorting of households as an outcome of migration choices, search frictions in the housing market, and housing bubbles. In one of the sensitivity analyses, we extend the model by allowing households to choose endogenously whether to migrate to Beijing, which generates a current price that is higher than in the baseline model but still substantially below the actual price in the data.

Based on the simulated data, we show that the empirical relations between housing price and fundamentals in a transition economy are time-varying. Therefore while regression analysis is useful for studying price-fundamentals relations in the developed markets, it can produce misleading results for transition economies where empirical relations between house price and economic fundamentals are not stable.

Since the model provides a structural link between price and fundamentals, it can be used to analyze the impacts of a number of policies, including the implementation of property tax, tightening of migration restrictions, and provision of universal medical insurance. We leave these analyses for future studies.

Appendices

A Proof of the *Proposition*

Given the exogenous growth factors of the fundamentals such as income, population and land supply, we will show that the growth factors of prices $\{G_p, G_r, G_q\}$ and the growth factors of the choice variables of the firm and households, as proposed in the ***Proposition***, satisfy the general equilibrium conditions. Specifically they are consistent with: (i) the optimization problem of the firm; (ii) the optimization problems of households; (iii) the market clearing conditions.

We prove the proposition by induction. Let K_t^*, L_t^* and H_t^* denote the firm's optimal decisions and $b_{a,t}^*$, $c_{a,t}^*$, $h_{a,t}^*$, $s_{a,t}^*$ denote a household's optimal decisions in period t. Further let p_t, r_t, and q_t be the market clearing house price, rent and land price. By definition, these choice variables and prices satisfy the general equilibrium conditions in period t. We will show that, using the proposed growth factors, $K_{t+1} = K^* G_K$, $L_{t+1} = L_t^* G_L$, $H_{t+1} = H_t^* G_H$, $b_{a,t+1} = b_{a,t}^* G_b$, $c_{a,t+1} = c_{a,t}^* G_c$, $h_{a,t+1} = h_{a,t}^* G_h$, $s_{a,t+1} = s_{a,t}^* G_s$, $p_{t+1} = p_t G_p$, $r_{t+1} = r_t G_r$ and $q_{t+1} = q_t G_q$ will also satisfy the three general equilibrium conditions in period $t+1$. In particular, we show that the decisions of households of age a at time $t+1$ are the same as the earlier cohort (those of age a at time t) up to the scale factors as proposed in the ***Proposition***, which guarantees that the average consumption and investment will grow at the proposed factors because the age distribution of households is time-invariant in the BGP.

Firm's Optimization Problem First we show that the growth factors are consistent with the firm's flow of funds equation:

$$p_t(H_t^* - H_{t-1}) = K_t^* - (1-\delta)K_{t-1} + q_t(L_t^* - L_{t-1}).$$

Assuming the equation above holds in period t, we want to show that its counterpart in period $t+1$ holds as well. We multiply both sides of the above equation with G_Y and apply the proposed growth factors. The left side of the equation is

$$\begin{aligned} p_t(H_t^* - H_{t-1})G_Y &= p_t(H_t^* - H_{t-1}) \left(\frac{G_Y}{G_L}\right)^{\theta} G_Y^{1-\theta} G_L^{\theta} \\ &= p_t(H_t^* - H_{t-1}) G_p G_H \\ &= p_t G_p (H_t^* G_H - H_{t-1} G_H) \\ &= p_{t+1}(H_{t+1}^* - H_t). \end{aligned}$$

The right side is

$$
\begin{aligned}
\left[K_t^* - (1-\delta)K_{t-1} + q_t(L_t^* - L_{t-1})\right] G_Y &= K_t^* G_Y - (1-\delta)K_{t-1}G_Y + q_t(L_t^* - L_{t-1})G_Y \\
&= K_t^* G_K - (1-\delta)K_{t-1}G_K + q_t(L_t^* - L_{t-1})\frac{G_Y}{G_L}G_L \\
&= K_{t+1}^* - (1-\delta)K_{t-1}G_K + q_t(L_t^* - L_{t-1})G_q G_L \\
&= K_{t+1}^* - (1-\delta)K_{t-1}G_K + q_{t+1}(L_{t+1}^* - L_{t-1}G_L).
\end{aligned}
$$

Therefore, given state variables $\{K_t, L_t\} = \{K_{t-1}G_K, L_{t-1}G_L\}$ we have

$$
p_{t+1}(H_{t+1}^* - H_t) = K_{t+1}^* - (1-\delta)K_t + q_t(L_{t+1}^* - L_t)
$$

That is, the flow of funds condition in period $t+1$ can be derived from that in period t and the growth factors in the ***Proposition***.

Next, we show the proposed growth factors are consistent with the firm's first-order condition with respect to K which is

$$
Z(1-\theta)\tilde{r}_t \left(\frac{K_t^*}{L_t^*}\right)^{-\theta} = 1 - \frac{1-\delta}{R_t} \tag{36}
$$

First of all, notice that in the BGP, $G_p = G_r$, therefore

$$
R_t = \frac{p_t + r_t}{p_{t-1}} = \frac{G_p(p_t + r_t)}{G_p p_{t-1}} = \frac{p_{t+1} + r_{t+1}}{p_t} = R_{t+1}.
$$

Thus housing return is a constant in the BGP, and $\tilde{r}_t = r_{t+1}/R_{t+1}$ grows at the same factor as r_{t+1}. Given (36), to prove the same first-order condition with respect to K holds in period $t+1$, it suffices to show $\tilde{r}_t(K_t^*/L_t^*)^{-\theta} = \tilde{r}_{t+1}(K_{t+1}^*/L_{t+1}^*)^{-\theta}$, which is straightforward using the proposed growth factors. Thus we omit the details and conclude that

$$
Z(1-\theta)\tilde{r}_{t+1} \left(\frac{K_{t+1}^*}{L_{t+1}^*}\right)^{-\theta} = 1 - \frac{1-\delta}{R_{t+1}}
$$

Third, we show the proposed growth factors are consistent with the firm's first-order condition with respect to L which is

$$
Z\theta\tilde{r}_t \left(\frac{K_t^*}{L_t^*}\right)^{1-\theta} = q_t - \frac{q_{t+1}}{R_t}
$$

We need to show that

$$
Z\theta\tilde{r}_{t+1} \left(\frac{K_t^* G_K}{L_t^* G_L}\right)^{1-\theta} = q_{t+1} - \frac{q_{t+2}}{R_{t+1}}
$$

Starting from the left-side of the equation above and substituting the proposed growth factors in the ***Proposition***, we have

$$
\begin{aligned}
Z\theta\tilde{r}_{t+1}\left(\frac{K_t^*G_K}{L_t^*G_L}\right)^{1-\theta} &= Z\theta\tilde{r}_tG_r\left(\frac{K_t^*}{L_t^*}\right)^{1-\theta}\left(\frac{G_K}{G_L}\right)^{1-\theta}\\
&= Z\theta\tilde{r}_t\left(\frac{G_L}{G_Y}\right)^{\theta}\left(\frac{K_t^*}{L_t^*}\right)^{1-\theta}\left(\frac{G_Y}{G_L}\right)^{1-\theta}\\
&= Z\theta\tilde{r}_t\left(\frac{K_t^*}{L_t^*}\right)^{1-\theta}\left(\frac{G_L}{G_Y}\right)^{\theta}\left(\frac{G_Y}{G_L}\right)^{1-\theta}\\
&= Z\theta\tilde{r}_t\left(\frac{K_t^*}{L_t^*}\right)^{1-\theta}\frac{G_Y}{G_L}\\
&= \left(q_t-\frac{q_{t+1}}{R_t}\right)G_q\\
&= q_{t+1}-\frac{q_{t+2}}{R_{t+1}}
\end{aligned}
$$

where the last equality holds because housing equity return is constant in the BGP.

Household's Optimization Problems Using the same strategy as with the firm's flow of fund equation, it is straightforward to show that the growth factors of $b_{a,t}$, $c_{a,t}$, $s_{a,t}$, $h_{a,t}$ are consistent with the households' budget constraint. The intra-temporal optimal allocation in household's problem is governed by equation (25). It is also straightforward to show that the proposed growth factors are consistent with this equation. For brevity, we omit the algebraic details which are available upon request.

To complete the proof that the proposed growth factors are consistent with household's optimization problems, we show that the functional equations that define V_{own}, V_{rent}, W_{own} and W_{rent} are all re-scalable so that if $b^*_{a,t}$, $c^*_{a,t}$, $h^*_{a,t}$, $s^*_{a,t}$ solve the optimization problems in period t, then $b_{a,t+1}=b^*_{a,t}G_b$, $c_{a,t+1}=c^*_{a,t}G_c$, $h_{a,t+1}=h^*_{a,t}G_h$, $s_{a,t+1}=s^*_{a,t}G_s$ will solve the optimization problems of households with the same age a in period $t+1$.

First, using the growth factors proposed the ***Proposition***, we show that both the bequest value and the utility function can be re-scaled by $\left(G_Y^{1-\theta\omega}G_L^{\theta\omega}/G_N\right)^{1-\gamma}$. From equation (17), in period $t+1$ we have

$$\begin{aligned} V_b(s_{a,t+1}) &= \mathcal{B}\left[(1-\omega)^{1-\omega}(\omega)^{\omega}\right]^{1-\gamma}\left(\frac{1}{r_{t+1}}\right)^{\omega(1-\gamma)}\frac{(p_{t+1}s_{a,t+1})^{1-\gamma}}{1-\gamma} \\ &= \mathcal{B}\left[(1-\omega)^{1-\omega}(\omega)^{\omega}\right]^{1-\gamma}\left(\frac{1}{r_t}\right)^{\omega(1-\gamma)}\frac{(p_t s_{a,t})^{1-\gamma}}{1-\gamma}\left(\frac{G_p G_s}{G_r^{\omega}}\right)^{1-\gamma} \\ &= V_b(s_{a,t})\left(\frac{G_p G_s}{G_r^{\omega}}\right)^{1-\gamma} \\ &= V_b(s_{a,t})\left(\frac{G_Y^{1-\theta\omega}G_L^{\theta\omega}}{G_N}\right)^{1-\gamma}, \end{aligned} \tag{37}$$

where we used the proposed growth factors for the last equality. For households whose bequest is in the risk-free asset, it is straightforward to show that $V_b(b_{a,t+1}) = V_b(b_{a,t})\left(\frac{G_Y^{1-\theta\omega}G_L^{\theta\omega}}{G_N}\right)^{1-\gamma}$.

Similarly, for the utility function we have

$$u(c_{a,t+1}, h_{a,t+1}) = u(c_{a,t}, h_{a,t})\left(\frac{G_Y^{1-\theta\omega}G_L^{\theta\omega}}{G_N}\right)^{1-\gamma}. \tag{38}$$

Finally, using equation (37) and (38), we can show via backward inductions that the household's value functions at any age can be re-scaled by $\left(G_Y^{1-\theta\omega}G_L^{\theta\omega}/G_N\right)^{1-\gamma}$. This property, combined with the fact that the household's budget constraints are consistent with the proposed growth factors, implies that the functional equations are re-scalable.[40]

Market Clearing We need to show that the proposed growth factors are consistent with the clearing of land market, housing consumption market and equity market. For the land market, it is sufficient to show that land demand grows at the same factor as the exogenous land supply G_L. The firm chooses an optimal land input in housing production given the land price, according to the first-order condition given by equation (7), which implies that

$$G_r\left(\frac{G_K}{G_L^d}\right)^{1-\theta} = G_q,$$

where G_L^d is the growth factor of land demand in the BGP. It follows that

$$G_L^d = \left(\frac{G_r}{G_q}\right)^{1/(1-\theta)} G_K. \tag{39}$$

[40]For technical reason, we assume that in the BGP, the minimum housing size grows at a constant factor of $(G_Y/G_N)^{1-\theta}(G_L/G_N)^{\theta}$, which is the same as the growth factor G_s of housing investment demand and the growth factor G_h of consumption demand. This assumption guarantees that the minimum housing size constraint is equally binding in each period in the BGP and helps to preserve the scalability of households' optimization problems.

Substituting the growth factors G_r, G_q, and G_K proposed in the ***Proposition***, we get $G_L^d = G_L$, hence the growth factors satisfy the land market clearing condition.

Next we show that the growth factors satisfy the housing consumption market clearing condition. Aggregate housing supply grows at a factor of G_H, while households' housing consumption demand grows at a factor of G_h. It is enough to show that $G_H/G_N = G_h$. By the housing production function (1), $G_H = G_K^{1-\theta} G_L^{\theta}$. Given $G_Y = G_K$ (see point 1 of the ***Proposition***), we have

$$\frac{G_H}{G_N} = \frac{G_Y^{1-\theta} G_L^{\theta}}{G_N} = \left(\frac{G_Y}{G_N}\right)^{1-\theta} \left(\frac{G_L}{G_N}\right)^{\theta} = G_h. \tag{40}$$

Therefore, these growth factors G_H and G_h as given in points 2-3 of the ***Proposition*** satisfy the housing consumption market clearing condition. The same argument applies to the home equity market clearing condition with G_h replaced by G_s.

FAR, Price-income Ratio and Price-rent ratio We have shown that growth factors in points 1-8 of the ***Proposition*** satisfy all the general equilibrium conditions. Using these growth factors, we have $FAR_{t+1}/FAR_t = \frac{H_{t+1}/L_{t+1}}{H_t/L_t} = \frac{G_H}{G_L} = \frac{G_Y^{1-\theta} G_L^{\theta}}{G_L} = (G_Y/G_L)^{1-\theta}$, which is point 9 of the ***Proposition*** . Also it is straightforward to show that both price-income ratio and price-rent ratio are time-invariant. Hence point 10 of the ***Proposition*** is true.

B Computation Strategy

Our model features heterogeneous agents and incomplete markets. House price, rent, land price and housing quantity of the economy in the BGP can be directly calculated from the terminal conditions discussed in details in Section 4.3.2. Below we explain how we numerically solve the model during the transition period.

A well-recognized challenge in heterogeneous agents model is that the distribution of assets used by economic agents to predict future prices is an infinite-dimension object which the computer cannot handle. In our case, households are heterogeneous in their asset holdings because they differ in age, history of income shocks and history of medical expense shocks. The distribution of assets among households affects the aggregate demand for housing consumption and investment, hence it matters for equilibrium house price and rent.

We are aware of two approaches to tackle this problem of infinite dimensions. The first approach, as exemplified in Krusell and Smith (1998), uses a few moments of the asset distribution to proxy the whole distribution, then uses certain function (typically an affine function) of these moments to predict future prices. The parameters of the affine function are so chosen that the predicted prices clear the market. An advantage of this approach lies in its capability of handling aggregate uncertainty – one just need to find a different set of parameters of the affine function for each aggregate state to predict the future price. In other words, the introduction of additional aggregate states does not increase the computation load exponentially. However for problems that involve a long transition period, this approach entails finding the appropriate parameters of the affine function in each period during the transition, because the mapping between the asset distribution and the future price could change substantially over time. In our case, the transition period spans 110 years, requiring the fitting of 110 affine functions. Thus this first approach does not work well for us.

The second approach guesses and verifies the paths of future prices without relying on the asset distribution. Since we are interested in the house price and rent paths rather than the mapping between asset distribution and the price or rent, the mapping can be ignored as long as the price and rent paths clear the markets. This approach works well in solving models featuring economy transition without aggregate uncertainty, like the model in this paper. But it is not a good choice if aggregate uncertainty exists. With aggregate uncertainty the price paths to guess and verify grow exponentially with the number of periods. For example, assume there is only one aggregate state variable that takes 2 possible outcomes, and the economic transition takes 110 periods. Then there are 2^{110} possible paths for the evolution of the aggregate state variable, hence the guess and verify strategy is too computationally extensive.

To implement the second approach, we use the following procedure to solve for housing supply, demand and market clearing paths of house price, rent and land price.

1. Guess a path of house price and a path of rent.
2. Compute the corresponding supply of and demand for housing consumption and housing equity at each point in time.
3. Derive the path of land price using equation (11).
4. Check the difference between supply and demand, and iterate the above steps until markets clear.

One major technical challenge we face is to find the market clearing prices and rents for one hundred and ten years, which is not quite feasible using standard search algorithms. Instead of relying on the mechanical updating schemes from the standard search algorithms, we update the price and rent paths toward the direction of balancing the supply and demand in the housing equity and rental markets under the model. The rental market is relatively easy to clear because supply and demand of rental housing are determined by current rent while future rents and house prices do not play any role. Thus our search algorithm simply increases the rental rates for any periods when demand exceeds supply, and vice versa.

To clear the housing equity market, it is important to take into account the current and future returns to equity investment which consist of dividend (rent) and capital gain (price appreciation). Suppose that demand exceeds supply between periods t and $t+j$, our search algorithm consists of the following three adjustments: (i) increasing price in period t; (ii) decreasing the growth rates of price between period t and $t+j$; (3) decreasing the rents between period t and $t+j$. These adjustments not only reduce demand for housing equity, but also increase the supply because they reduce the financing cost of the firm, as discussed in Section 3.1.4

We approximate the paths of house price and rent as functions of time, using six-order polynomials. Since there is no aggregate uncertainty in the model, both price and rent paths are smooth. Therefore polynomial approximations work very well. Consequently, the updating of price path and rent path is simplified to the updating of the polynomial coefficients of the price path and rent path respectively.

In practice, we start with an initial guess of price and rent paths that are increasing in time, then solve for the supply of and demand for housing equity. Next we update the path of house price based on the supply and demand, and then update the path of rent based on the supply and demand in the rental market. We use large-scale updating in the beginning, then gradually reduce the size of updating, until the paths of supply and demand converge.

C Data on House Price and Rent

In this paper we make use of market data in two ways. First, we compare the model-implied house price in 2014 to the observed average house price in 2014 for the Beijing market. Second, we compare the price-income and price-rent ratios under the calibrated model to the historical ratios observed in the 2005 to 2013 period.

To ensure proper mapping between the data and the model, we obtain the market house price for Beijing in 2014 as the average house price of both newly-built and existing homes without quality adjustment. Specifically, following Garriga et al. (2016) and others, we obtain the average sales price for new homes from the National Bureau of Statistics (NBS). Since the NBS does not provide the price level for existing homes, we obtain the 2014 existing home price from the WIND database – a well-known database that collects information from financial and real estate markets.[41] The average house sales price in both NBS and WINDs are quality unadjusted, and calculated as total dollar sales of residential buildings divided by the floor space of residential buildings. We then impute the weighted average of the new home price and the existing home price, using the shares of new and existing home transactions in the total value of transactions in Beijing in 2014 as the weights.[42] As shown in Table C, for the Beijing market in 2014, the new home price reported by NBS is 18,499 RMB per square meter, and the existing home price is 34,106 RMB per square meter according to the WIND database. The weight on new homes is 37.9% in 2014. We take the weighted average number, 28,194 RMB, as the Beijing house price in 2014 (in terms of 2014 RMB).

Due to the data limitation, we do not always observe average house price in the years prior to 2014, so we use the new (resp. resale) house price level in 2014 obtained above, combined with the NBS new home price index (resp. the NBS resale home price index), to infer the nominal price for new (resp. resale) homes in earlier years, and then use CPI to convert nominal prices into real prices in terms of 2014 RMB. Finally, for each year, we impute the weighted average of new home price and resale home price, using the shares of new and resale home transactions in that year as weights. We label the Beijing house price series obtained this way as the "NBS" series in Table C.

Although appealing, the NBS price series have some limitations due to their inconsistent sampling method over time and lack of control for house quality, as pointed out by Fang et al. (2016) and Wu et al. (2016) among others. For these reasons, we provide an additional price measure as a robustness check. We name the new measure as the "THU" series because it is based on the constant quality house price indices provided by the Hang Lung Center for Real Estate at Tsinghua University. The center provides price indices for new homes (since 2004) and price indices for resale homes (since 2009).[43] The THU series we imputed

[41] See `http://www.wind.com.cn/`. Although NBS does not not provide the price level for existing homes, it provides a price index for existing homes, as well as a price index for new homes. Later, we use both price indices when measuring market prices in years prior to 2014.

[42] We thank Jing Wu for generously providing the transaction shares of the new and existing homes over time. The original data are recorded in the Real Estate Market Information System (REMIS) administered by the municipal housing authority in Beijing.

[43] The data are publicly available at `http://www.tsinghua.edu.cn/publish/creen/9569/index.html`. There are three different measures of price indices: average sales price, hedonic index and repeated-sales index (see http://www.cre.tsinghua.edu.cn/publish/cre/9254/20150505/15251430808272951.pdf).

uses the repeated sales index which does not adjust for land supply, floor-to-area ratio and suburbanization (through distance to city centre). In an ideal world where housing stock is truly homogenous, the repeated sales index would be equivalent to the average-price-based NBS index. By controlling for quality variations, the THU price measure reflects the average price of constant-quality homes in Beijing over time, consistent with the "homogenous housing stock" concept of our theoretic model.

Table C also presents the annual house rent series based on the rental housing price index for Beijing as reported by the NBS. The NBS estimates the rent series using the rents of both new and existing homes. The average annual rent is 744 RMB per square meter in 2014.[44]

To obtain conceptually consistent measures price-income (price-rent) ratio as in the model, we use the weighted average price (rent) of existing and new homes reported by the NBS. We define price-income ratio as *(price per square meter)×(average number of square meters per capita)/(average income per capita).* Data on income and housing size per capita are also from the NBS. The last column of Table C shows housing size per capita in Beijing which has been increasing steadily. Price-rent ratio is calculated as *(average house price) / (average rent).*

[44] Rental housing price index is available at `http://data.stats.gov.cn/easyquery.htm?cn=E0103&zb=A0902&reg=110000&sj=2013`. The 2014 rent level is from the Guoxinda database `http://www.gxdgroup.com.cn/`.

Table C: House price, rent and housing size per capita

	Weights		NBS Series (2014 RMB)			THU Series (2014 RMB)			Rent (2014 RMB)	Size (m^2)
	existing	new	existing	new	overall	existing	new	overall		
2005	0.26	0.74	28478	7997	13411				593.90	25.9
2006	0.34	0.66	33889	9430	17797				602.81	26.5
2007	0.44	0.56	34621	13044	22567				608.23	27.1
2008	0.53	0.47	32561	13493	23692				622.83	27.9
2009	0.60	0.40	33567	15461	26266	14158	8721	11966	654.59	28.8
2010	0.71	0.29	33276	19430	29263	22626	13199	19894	644.78	28.9
2011	0.69	0.31	31143	16695	26684	24831	13851	21442	660.25	29.1
2012	0.62	0.38	31576	17341	26101	24673	14611	20803	697.22	29.3
2013	0.69	0.31	35249	18229	30044	34311	17063	29037	720.23	31.3
2014	0.62	0.38	34106	18499	28194	34106	18499	28194	744.00	31.5

Note: This table shows the weights of new and existing houses, house price, annual rent and housing size per capita in Beijing between 2005-2014. The weights are the fraction of new and existing home transactions in the total value of home transactions in a given a year.

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