Online Appendix for Housing Price and Fundamentals in a Transition Economy: The Case of the Beijing Market

Bing Han, Lu Han, and Guozhong Zhu*

In this online companion to the main paper, we provide additional materials containing the proofs, details about the computational method we develop to solve the model, as well as the projection of economic fundamentals and calibration of model parameters. We also report results of further robustness checks, including under an extended model featuring labor as an additional input in the housing production function.

1 Proof of the *Proposition* about Properties of BGP

Given the exogenous growth factors of the fundamentals such as income, population and land supply, we will show that the growth factors of prices $\{G_p, G_r, G_q\}$ and the growth factors of the choice variables of the firm and households, as proposed in the ***Proposition***, satisfy the general equilibrium conditions. Specifically they are consistent with: (i) the optimization problem of the firm; (ii) the optimization problems of households; (iii) the market clearing conditions.

We prove the proposition by induction. Let K_t^*, L_t^* and H_t^* denote the firm's optimal decisions and $b_{a,t}^*$, $c_{a,t}^*$, $h_{a,t}^*$, $s_{a,t}^*$ denote a household's optimal decisions in period t. Further let p_t, r_t, and q_t be the market clearing house price, rent and land price. By definition, these choice variables and prices satisfy the general equilibrium conditions in period t. We will show that, using the proposed growth factors, $K_{t+1} = K^* G_K$, $L_{t+1} = L_t^* G_L$, $H_{t+1} = H_t^* G_H$,

*Bing Han, Rotman School of Management, University of Toronto, 105 St. George Street, Toronto, Ontario, Canada M5S3E6, Email: Bing.Han@Rotman.Utoronto.Ca, Tel: (416) 946-0732. Lu Han, Rotman School of Management, University of Toronto, 105 St. George Street, Toronto, Ontario, Canada M5S3E6, Email: Lu.Han@Rotman.Utoronto.Ca, Tel: (416) 946-5294. Guozhong Zhu, Alberta School of Business, University of Alberta, Email: guozhong@ualberta.ca, Tel: 780-492-2426.

$b_{a,t+1} = b^*_{a,t}G_b$, $c_{a,t+1} = c^*_{a,t}G_c$, $h_{a,t+1} = h^*_{a,t}G_h$, $s_{a,t+1} = s^*_{a,t}G_s$, $p_{t+1} = p_tG_p$, $r_{t+1} = r_tG_r$ and $q_{t+1} = q_tG_q$ will also satisfy the three general equilibrium conditions in period $t+1$. In particular, we show that the decisions of households of age a at time $t+1$ are the same as the earlier cohort (those of age a at time t) up to the scale factors as proposed in the ***Proposition***, which guarantees that the average consumption and investment will grow at the proposed factors because the age distribution of households is time-invariant in the BGP.

Firm's Optimization Problem First we show that the growth factors are consistent with the firm's flow of funds equation:

$$p_t(H^*_t - H_{t-1}) = K^*_t - (1-\delta)K_{t-1} + q_t(L^*_t - L_{t-1}).$$

Assuming the equation above holds in period t, we want to show that its counterpart in period $t+1$ holds as well. We multiply both sides of the above equation with G_Y and apply the proposed growth factors. The left side of the equation is

$$\begin{aligned}
p_t(H^*_t - H_{t-1})G_Y &= p_t(H^*_t - H_{t-1})\left(\frac{G_Y}{G_L}\right)^{\theta} G_Y^{1-\theta}G_L^{\theta} \\
&= p_t(H^*_t - H_{t-1})G_pG_H \\
&= p_tG_p(H^*_tG_H - H_{t-1}G_H) \\
&= p_{t+1}(H^*_{t+1} - H_t).
\end{aligned}$$

The right side is

$$\begin{aligned}
\left[K^*_t - (1-\delta)K_{t-1} + q_t(L^*_t - L_{t-1})\right]G_Y &= K^*_tG_Y - (1-\delta)K_{t-1}G_Y + q_t(L^*_t - L_{t-1})G_Y \\
&= K^*_tG_K - (1-\delta)K_{t-1}G_K + q_t(L^*_t - L_{t-1})\frac{G_Y}{G_L}G_L \\
&= K^*_{t+1} - (1-\delta)K_{t-1}G_K + q_t(L^*_t - L_{t-1})G_qG_L \\
&= K^*_{t+1} - (1-\delta)K_{t-1}G_K + q_{t+1}(L^*_{t+1} - L_{t-1}G_L).
\end{aligned}$$

Therefore, given state variables $\{K_t, L_t\} = \{K_{t-1}G_K, L_{t-1}G_L\}$ we have

$$p_{t+1}(H^*_{t+1} - H_t) = K^*_{t+1} - (1-\delta)K_t + q_t(L^*_{t+1} - L_t)$$

That is, the flow of funds condition in period $t+1$ can be derived from that in period t and the growth factors in the ***Proposition***.

Next, we show the proposed growth factors are consistent with the firm's first-order condition with respect to K which is

$$Z(1-\theta)\tilde{r}_t\left(\frac{K^*_t}{L^*_t}\right)^{-\theta} = 1 - \frac{1-\delta}{R_t} \tag{1}$$

First of all, notice that in the BGP, $G_p = G_r$, therefore

$$R_t = \frac{p_t + r_t}{p_{t-1}} = \frac{G_p(p_t + r_t)}{G_p p_{t-1}} = \frac{p_{t+1} + r_{t+1}}{p_t} = R_{t+1}.$$

Thus housing return is a constant in the BGP, and $\tilde{r}_t = r_{t+1}/R_{t+1}$ grows at the same factor as r_{t+1}. Given (1), to prove the same first-order condition with respect to K holds in period $t+1$, it suffices to show $\tilde{r}_t(K_t^*/L_t^*)^{-\theta} = \tilde{r}_{t+1}(K_{t+1}^*/L_{t+1}^*)^{-\theta}$, which is straightforward using the proposed growth factors. Thus we omit the details and conclude that

$$Z(1-\theta)\tilde{r}_{t+1}\left(\frac{K_{t+1}^*}{L_{t+1}^*}\right)^{-\theta} = 1 - \frac{1-\delta}{R_{t+1}}$$

Third, we show the proposed growth factors are consistent with the firm's first-order condition with respect to L which is

$$Z\theta\tilde{r}_t\left(\frac{K_t^*}{L_t^*}\right)^{1-\theta} = q_t - \frac{q_{t+1}}{R_t}$$

We need to show that

$$Z\theta\tilde{r}_{t+1}\left(\frac{K_t^* G_K}{L_t^* G_L}\right)^{1-\theta} = q_{t+1} - \frac{q_{t+2}}{R_{t+1}}$$

Starting from the left-side of the equation above and substituting the proposed growth factors in the ***Proposition***, we have

$$\begin{aligned}
Z\theta\tilde{r}_{t+1}\left(\frac{K_t^* G_K}{L_t^* G_L}\right)^{1-\theta} &= Z\theta\tilde{r}_t G_r\left(\frac{K_t^*}{L_t^*}\right)^{1-\theta}\left(\frac{G_K}{G_L}\right)^{1-\theta} \\
&= Z\theta\tilde{r}_t\left(\frac{G_L}{G_Y}\right)^{\theta}\left(\frac{K_t^*}{L_t^*}\right)^{1-\theta}\left(\frac{G_Y}{G_L}\right)^{1-\theta} \\
&= Z\theta\tilde{r}_t\left(\frac{K_t^*}{L_t^*}\right)^{1-\theta}\left(\frac{G_L}{G_Y}\right)^{\theta}\left(\frac{G_Y}{G_L}\right)^{1-\theta} \\
&= Z\theta\tilde{r}_t\left(\frac{K_t^*}{L_t^*}\right)^{1-\theta}\frac{G_Y}{G_L} \\
&= \left(q_t - \frac{q_{t+1}}{R_t}\right)G_q \\
&= q_{t+1} - \frac{q_{t+2}}{R_{t+1}}
\end{aligned}$$

where the last equality holds because housing equity return is constant in the BGP.

Household's Optimization Problems Using the same strategy as with the firm's flow of fund equation, it is straightforward to show that the growth factors of $b_{a,t}$, $c_{a,t}$, $s_{a,t}$, $h_{a,t}$

are consistent with the households' budget constraint. It is also straightforward to show that the proposed growth factors are consistent with this equation. For brevity, we omit the algebraic details which are available upon request.

To complete the proof that the proposed growth factors are consistent with household's optimization problems, we show that the functional equations that define V_{own}, V_{rent}, W_{own} and W_{rent} are all re-scalable so that if $b^*_{a,t}$, $c^*_{a,t}$, $h^*_{a,t}$, $s^*_{a,t}$ solve the optimization problems in period t, then $b_{a,t+1} = b^*_{a,t}G_b$, $c_{a,t+1} = c^*_{a,t}G_c$, $h_{a,t+1} = h^*_{a,t}G_h$, $s_{a,t+1} = s^*_{a,t}G_s$ will solve the optimization problems of households with the same age a in period $t+1$.

First, using the growth factors proposed the ***Proposition***, we show that both the bequest value and the utility function can be re-scaled by $\left(G_Y^{1-\theta\omega}G_L^{\theta\omega}/G_N\right)^{1-\gamma}$. From equation (12) in the main body of the paper, in period $t+1$ we have

$$\begin{aligned} V_b(s_{a,t+1}) &= \mathcal{B}\left[(1-\omega)^{1-\omega}(\omega)^{\omega}\right]^{1-\gamma}\left(\frac{1}{r_{t+1}}\right)^{\omega(1-\gamma)}\frac{(p_{t+1}s_{a,t+1})^{1-\gamma}}{1-\gamma} \\ &= \mathcal{B}\left[(1-\omega)^{1-\omega}(\omega)^{\omega}\right]^{1-\gamma}\left(\frac{1}{r_t}\right)^{\omega(1-\gamma)}\frac{(p_t s_{a,t})^{1-\gamma}}{1-\gamma}\left(\frac{G_pG_s}{G_r^{\omega}}\right)^{1-\gamma} \\ &= V_b(s_{a,t})\left(\frac{G_pG_s}{G_r^{\omega}}\right)^{1-\gamma} \\ &= V_b(s_{a,t})\left(\frac{G_Y^{1-\theta\omega}G_L^{\theta\omega}}{G_N}\right)^{1-\gamma}, \end{aligned} \tag{2}$$

where we used the proposed growth factors for the last equality. For households whose bequest is in the risk-free asset, it is straightforward to show that $V_b(b_{a,t+1}) = V_b(b_{a,t})\left(\frac{G_Y^{1-\theta\omega}G_L^{\theta\omega}}{G_N}\right)^{1-\gamma}$.

Similarly, for the utility function we have

$$u(c_{a,t+1}, h_{a,t+1}) = u(c_{a,t}, h_{a,t})\left(\frac{G_Y^{1-\theta\omega}G_L^{\theta\omega}}{G_N}\right)^{1-\gamma}. \tag{3}$$

Finally, using equation (2) and (3), we can show via backward inductions that the household's value functions at any age can be re-scaled by $\left(G_Y^{1-\theta\omega}G_L^{\theta\omega}/G_N\right)^{1-\gamma}$. This property, combined with the fact that the household's budget constraints are consistent with the proposed growth factors, implies that the functional equations are re-scalable.[1]

[1]For technical reason, we assume that in the BGP, the minimum housing size grows at a constant factor of $(G_Y/G_N)^{1-\theta}(G_L/G_N)^{\theta}$, which is the same as the growth factor G_s of housing investment demand and the growth factor G_h of consumption demand. This assumption guarantees that the minimum housing size constraint is equally binding in each period in the BGP and helps to preserve the scalability of households' optimization problems.

Market Clearing We need to show that the proposed growth factors are consistent with the clearing of land market, housing consumption market and equity market. For the land market, it is sufficient to show that land demand grows at the same factor as the exogenous land supply G_L. The firm chooses an optimal land input in housing production given the land price, according to the first-order condition given by equation (6) in the main body of the paper, which implies that

$$G_r \left(\frac{G_K}{G_L^d}\right)^{1-\theta} = G_q,$$

where G_L^d is the growth factor of land demand in the BGP. It follows that

$$G_L^d = \left(\frac{G_r}{G_q}\right)^{1/(1-\theta)} G_K. \tag{4}$$

Substituting the growth factors G_r, G_q, and G_K proposed in the ***Proposition***, we get $G_L^d = G_L$, hence the growth factors satisfy the land market clearing condition.

Next we show that the growth factors satisfy the housing consumption market clearing condition. Aggregate housing supply grows at a factor of G_H, while households' housing consumption demand grows at a factor of G_h. It is enough to show that $G_H/G_N = G_h$. By the housing production function, $G_H = G_K^{1-\theta} G_L^{\theta}$. Given $G_Y = G_K$ (see point 1 of the ***Proposition***), we have

$$\frac{G_H}{G_N} = \frac{G_Y^{1-\theta} G_L^{\theta}}{G_N} = \left(\frac{G_Y}{G_N}\right)^{1-\theta} \left(\frac{G_L}{G_N}\right)^{\theta} = G_h. \tag{5}$$

Therefore, these growth factors G_H and G_h as given in points 2-3 of the ***Proposition*** satisfy the housing consumption market clearing condition. The same argument applies to the home equity market clearing condition with G_h replaced by G_s.

FAR, Price-income Ratio and Price-rent ratio We have shown that growth factors in points 1-8 of the ***Proposition*** satisfy all the general equilibrium conditions. Using these growth factors, we have $FAR_{t+1}/FAR_t = \frac{H_{t+1}/L_{t+1}}{H_t/L_t} = \frac{G_H}{G_L} = \frac{G_Y^{1-\theta} G_L^{\theta}}{G_L} = (G_Y/G_L)^{1-\theta}$, which is point 9 of the ***Proposition*** . Also it is straightforward to show that both price-income ratio and price-rent ratio are time-invariant. Hence point 10 of the ***Proposition*** is true.

2 Model Solving and Computation Strategy

Our dynamic equilibrium model is difficult to solve due to two model features. First, because of uninsurable idiosyncratic income and medical expense shocks, the model features an incomplete market, and does not admit of an analytical solution along the transition path. Second, our model features heterogeneous agents. A well-recognized challenge in heterogeneous agents model is that the distribution of assets used by economic agents to predict future prices is an infinite-dimension object which the computer cannot handle. In our case, households are heterogeneous in their asset holdings because they differ in age, history of income shocks and medical expense shocks. We develop an efficient method to solve the dynamic equilibrium numerically using backward induction with the properties of the BGP as the terminal conditions.

It is important to note that in solving the model equilibrium during the economic transition periods, we do not rely on any historical data for the Beijing housing market (such as recent housing prices). We only use historical data about fundamental variables in Beijing to estimate their dynamics and then project their future evolutions. In addition, we need market data from a reference city for the terminal conditions. We calibrate the model parameters so that several key properties of the Beijing housing market in the BGP resemble those computed based on recent data from the reference city. Given the projected exogenous fundamentals and a set of calibrated model parameters, we endogenously solve the paths of house price, rent, and land price that clear the housing investment market, the housing consumption market and the land market.

An alternative approach is to first use historical data from the Beijing housing market to fit some reduced-form model of housing price as a function of the fundamentals and then extrapolate future house price based on the estimated relationship and the projection of the fundamentals. But this approach is not suitable for a transition economy where the relationship between house price and fundamentals is complex and unstable. In contrast, our approach is based on a dynamic equilibrium model without relying on historical house price data. Therefore, our approach not only circumvents non-stationarity encountered in a transition economy, but also shed light on how the equilibrium prices change in response to the changes in the fundamental variables.

A well-recognized challenge in heterogeneous agents model is that the distribution of assets used by economic agents to predict future prices is an infinite-dimension object which the computer cannot handle. In our case, households are heterogeneous in their asset holdings because they differ in age, history of income shocks and medical expense shocks.

We are aware of two approaches to tackle this problem of infinite dimensions. The first approach, as exemplified in Krusell and Smith (1998), uses a few moments of the asset distribution to proxy the whole distribution, then uses certain function (typically an affine function) of these moments to predict future prices. The parameters of the affine function are so chosen that the predicted prices clear the market. An advantage of this approach lies in its capability of handling aggregate uncertainty – one just need to find a different set of parameters of the affine function for each aggregate state to predict the future price. However for problems that involve a long transition period, this approach entails finding the appropriate parameters of the affine function in each period during the transition, because the mapping between the asset distribution and the future price could change substantially over time. In our case, the transition period spans 110 years, requiring the fitting of 110 affine functions. Thus this first approach does not work well for us.

The second approach guesses and verifies the paths of future prices without relying on the asset distribution. Since we are interested in the house price and rent paths rather than the mapping between asset distribution and the price or rent, the mapping can be ignored as long as the price and rent paths clear the markets. This approach works well in solving models featuring economy transition without aggregate uncertainty. But it is not a good choice if the model includes aggregate uncertainty which cause the price paths grow exponentially with the number of periods. For example, assume there is only one aggregate state variable that takes 2 possible outcomes, and the economic transition takes 110 periods. Then there are 2^{110} possible paths for the evolution of the aggregate state variable, hence the guess and verify strategy is too computationally extensive.

To implement the second approach, we use the following procedure to solve for housing supply, demand and market clearing paths of house price, rent and land price.

1. Guess a path of house price and a path of rent.
2. Compute the corresponding supply of and demand for housing consumption and housing equity at each point in time.
3. Derive the paths of land price using the path of house price and rent.
4. Check the difference between supply and demand, and iterate the above steps until markets clear.

One major technical challenge we face is to find the market clearing prices and rents for one hundred and ten years, which is not quite feasible using standard search algorithms. Instead of relying on the mechanical updating schemes from the standard search algorithms, we update the price and rent paths toward the direction of balancing the supply and demand in the housing equity and rental markets under the model.

The rental market is relatively easy to clear because supply and demand of rental housing are determined by current rent while future rents and house prices do not play any role. Thus our search algorithm simply increases the rental rates for any periods when demand exceeds supply, and vice versa.

To clear the housing equity market, it is important to take into account the current and future returns to equity investment which consist of dividend (rent) and capital gain (price appreciation). Suppose that demand exceeds supply between periods t and $t+j$, our search algorithm consists of the following three adjustments: (i) increasing price in period t; (ii) decreasing the growth rates of price between period t and $t+j$; (3) decreasing the rents between period t and $t+j$. These adjustments not only reduce demand for housing equity, but also increase the supply because they reduce the financing cost of the firm.

We approximate the paths of house price and rent as functions of time, using six-order polynomials. Since there is no aggregate uncertainty in the model, both price and rent paths are smooth. Therefore polynomial approximations work very well. Consequently, the updating of price path and rent path is simplified to the updating of the polynomial coefficients of the price path and rent path respectively.

In practice, we start with an initial guess of price and rent paths that are increasing in time, then solve for the supply of and demand for housing equity. Next we update the path of house price based on the supply and demand, and then update the path of rent based on the supply and demand in the rental market. We use large-scale updating in the beginning, then gradually reduce the size of updating, until the paths of supply and demand converge.

3 Projection of Fundamentals

This section describes our projection regarding population structure, land supply and aggregate income.

3.1 Evolution of Population Structure

Two dimensions of the population structure are relevant for our model: the population size and the age distribution.

The population data are obtained from the 2010 Census, and from *Sample Survey on Population Change* for other years between 2005 and 2013. The upper-left panel of Figure 1 plots the age distribution of Beijing residents, defined as individuals who either have formal registration (Hukou) or live in Beijing for more than six months per year. Compared with the overall urban population, Beijing population is much younger due to the influx of young migrants.

To project the population structure after 2013, we need to predict the fertility rate, the mortality rate and the immigration rate of the Beijing population. Given the recent relaxation of the one-child policy in China, we assume that the fertility rate switches from an "old rate" regime to a "new rate" regime in 2024. For the "old rate" regime, we use the data between 2005 and 2013 to calculate the age-specific fertility rate and mortality rate of Beijing population. If this trend continues, it would imply an ever decreasing total population in the future. For the "new rate" regime, we assume that beginning in 2024, the fertility trend rises linearly for the next 10 years so that the overall population growth rate reaches 0.4% in 2034 and then fertility rate is assumed to be time-invariant afterwards. Both trends are presented in the upper-right panel of Figure 1. Given that households have perfect foresight, the switch of the regimes is taken with full certainty. The mortality rate is assumed to be constant over time since life expectancy in China is already close to that of industrialized countries.

As emphasized in Henderson (2010), rapid urbanization is one of the key issues in population dynamics for a transition economy. Urbanization is reflected in the migration of households to Beijing in our model. At the end of 2014, Beijing permanent residents are comprised of 13.5 million local residents with Hukou and another 8.02 million "resident alien population" without Hukou – migrants who move from elsewhere in China (rural or other urban cities) and live in Beijing for more than six months per year. The migration rate, or the number of new migrants as a fraction of existing Beijing population, has been declining since 2008, and averages around 2.88% between 2010 and 2014.[2] In the baseline model, we assume that the migration rate decreases linearly from 2.88% in 2014 to zero by 2044. In the data it is clear that migrants to Beijing are mainly young workers; therefore we assume that only those aged between 21 and 30 migrate to Beijing in each year.

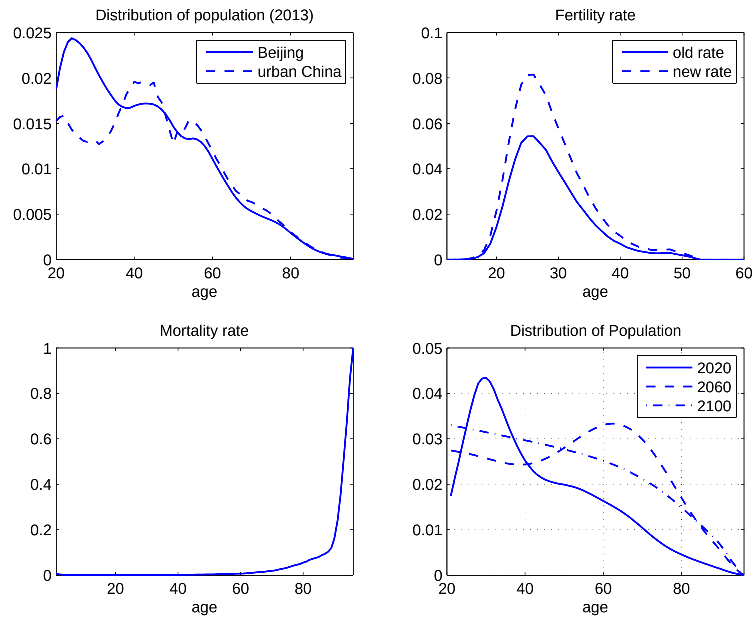
Based on the 2013 data and using the fertility rate, mortality rate and immigration rate discussed above, we extrapolate the population structure after 2013. The lower-right panel of Figure 1 plots the projected age structure of population in 2020, 2060 and 2100.

3.2 Evolution of Land Supply

In China, local governments own land and auction land use right. The amount of land to be auctioned depends on a multitude of considerations, including policies from the central

[2]According to the Beijing Municipal Bureau of Statistics, Beijing has another estimated 7.5 million floating population at the end of 2013. Given the short-term nature of these migrants, we leave them out in the calculation of migration rate.

Figure 1: Fertility rate, mortality rate, and population structure



This figure shows the initial age distribution of population, fertility rate, mortality rate and projected population structure. The fertility rate is presented as one half of women's fertility rate in the data, interpreted as fertility rate per couple. The "Old Rate" is the average fertility rate between 2005 and 2013 in the data. The "New Rate" is the projected fertility rate after 2034.

government, the fiscal balance of the local governments and the growth rate of the local GDP. Our model takes land supply as exogenously given, but land price and housing supply are both determined endogenously.

We obtain data on the supply of residential land from the National Bureau of Statistics (NBS). For each of the major cities in China, NBS reports the amount of new residential land acquired by housing developers.[3] This is the flow of land. The stock of land in 2009 is available from the 2010 *China Statistical Year Book of Environment* complied by NBS. Table 11-3 of the year book is "Basic Statistics on Urban Area and Land Used for Construction by Region" which reports that the area of residential land was 38,330 hectares in Beijing at the end of 2009. Based on the stock of land in 2009 and the annual flows, we obtain the historical total stock of residential land in Beijing, and then divide it by the population of Beijing residents to obtain land supply per capita. From Table 1, it is clear that in Beijing the total land stock growth has significantly lagged the growth of population since 2005,

[3] `http://data.stats.gov.cn/workspace/index?m=csnd`.

leading to a declining land supply per capita.

Table 1: Land and population of urban Beijing

	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
Population (million)	13.9	14.2	14.6	14.9	15.4	16.0	16.8	17.7	18.6	19.6	20.2	20.7	21.4
Growth rate (%)	–	2.75	2.33	2.49	3.03	4.10	4.68	5.67	5.03	5.48	2.89	2.51	3.61
New land (ha)	1472	2093	1391	1572	774	295	392	823	625	859	507	306	906
Total land stock (ha)	29518	30990	33082	34474	36046	36820	37115	37507	38330	38955	39814	40321	40627
Growth rate (%)	–	4.99	6.75	4.21	4.56	2.15	0.80	1.05	2.20	1.63	2.20	1.27	0.76
Land per capita (m^2)	21.31	21.77	22.72	23.09	23.44	23.00	22.14	21.18	20.61	19.86	19.72	19.49	18.95
Growth rate (%)	–	2.18	4.32	1.67	1.48	-1.87	-3.71	-4.37	-2.69	-3.65	-0.67	-1.21	-2.75

We assume that the land supply grows at a constant rate of 0.5% from 2014 onwards in the baseline model. Land supply per capita falls gradually due to the inflow of migrants during the urbanization process. As the growth of population plateaus, land supply per capita becomes time-invariant. In the sensitivity analysis we also consider cases where the growth of aggregate land supply is either 1% or 0%.

3.3 Evolution of Aggregate Income

The average disposable income of Beijing residents, as reported by the NBS, is 38.17, 41.13, and 43.91 thousand Renminbi (RMB) in year 2012, 2013 and 2014 respectively, each in terms of 2014 RMB.[4] These are roughly one-third of the disposable income of Hong Kong residents in the corresponding years. We assume that average income grows at a constant rate of 7% in 2015, then the growth rate declines linearly from 7% to 3% over the next 30 years. Given the assumption that population grows at a constant rate of 0.4% in the BGP, the growth rate of average income is $(1 + 3\%)/(1 + 0.4\%) - 1 = 2.59\%$ per year. In the sensitivity analysis, we consider two alternative cases where income growth plateaus to 3% after either 20 or 40 years.

[4]Each year the NBS reports income based on a random sample of households in Beijing. The sample includes two major components: residents with Beijing Hukou and residents without Beijing Hukou but living in Beijing for over 6 months every year.

4 Calibration of Model Parameters

4.1 Reference City and Empirical Moments

In the baseline specification, we assume that Beijing housing market in the BGP will resemble the current state of Hong Kong in terms of price-income ratio, price-rent ratio and growth rate of real house price. The Hong Kong Rating and Valuation Department reports data for different housing classes in the three main regions of Hong Kong – Hong Kong island, Kowloon and New Territory. Averaging over different classes in the three regions, the annual housing rent in Hong Kong is about 3.39 thousand HK dollars per square meter in 2012. Using an exchange rate of 0.82 RMB per HK dollar in 2012, the annual rent is about 2.78 thousand RMB per square meter. The average house price is 121 thousand HK dollars in 2012 which is about 99 thousand RMB.

Price-rent ratio is calculated as *(average house price) / (average rent)* $= 99/2.78 \approx 35.6$. We validate this ratio by calculating the ratio for each class of housing in each of the three regions in 2012, then taking the average of these ratios. The resulting number is 34.8. Price-income ratio is calculated as *(price per square meter)*$\times$*(average number of square meters per capita)/(average income per capita)*. Based on the 2012 data, housing per capita is about 12 square meters and average disposable income per capita is around 100 thousand RMB in Hong Kong. Therefore the price-income ratio is 11.88.

The growth rate of real house price in Hong Kong is calculated from house price index and CPI; the former is available from the Hong Kong Rating and Valuation Department, and the latter is available from the Census and Statistics Department. During the period of 1981-2012, the geometric mean of the growth rate of house price in Hong Kong, adjusted for inflation, is 2.14%. Thus, we take the growth factor of house price for Beijing in the BGP to be $G_p = 1.0214$.

Real housing return is calculated from the price-rent ratio and the real appreciation of house price. Given the price-rent ratio of 35.6 and the real appreciation rate of house price of 2.14%, return on housing investment is $R = 1.0495$, which we take as the return on housing investment for Beijing in the BGP.

4.2 Parameters Related to Housing Production

Three parameters are related to housing production: land share in production (θ), capital depreciation rate (δ) and scaling parameter in housing production (Z). We pin down θ by comparing the growth rate of house price to the growth rate of land price. As is evident from the ***Proposition***, in the BGP, the growth factors of house price (G_p) and land price (G_q) satisfy $G_p = G_q^\theta$, therefore

$$\theta = \frac{logG_p}{logG_q} \tag{6}$$

We calculate the growth factors based on Hong Kong data between 1987 and 2012. During this period, θ implied by the growth rates of house price and land price is 0.839, 0.497 and 0.819 for Hong Kong island, Kowloon and New Territory respectively. We take the average value $\theta = 0.72$.

In our model, the depreciation of housing stock is captured by the depreciation of capital (δ). Leigh (1980) estimates that the annual depreciation rate of housing in the United States is between 0.0036 and 0.0136. Letting δ^h denote this housing depreciation rate, then capital depreciation rate is $(1 - \delta^h)^{1/(1-\theta)}$ based on our housing production function. Therefore capital depreciation should be between 0.01 and 0.03. We use $\delta = 0.02$ in all quantitative analyses.

To pin down the scaling parameter Z, we use the housing supply equation. Since price-income ratio (pH/Y) and price-rent ratio (p/r) are constants in the BGP, housing supply per capita H in the BGP satisfies

$$H = \frac{ratio^{py}}{ratio^{pr}} \times \frac{Y}{r} \tag{7}$$

where $ratio^{py}$ and $ratio^{pr}$ are price-income ratio and price-rent ratio respectively, Y is the average income per capita and r is the housing rent. Substituting equation (7) into the housing supply equation yields

$$Z = \left(\frac{1}{r}\right) \left(\frac{ratio^{py}}{ratio^{pr}} \times \frac{Y}{L}\right)^{\theta} \left[\frac{1-(1-\delta)/R}{1-\theta}\right]^{1-\theta}. \tag{8}$$

In Hong Kong, housing per capita is about 12 square meters and the average floor-area ratio is about 4.5, thus land use per capita is $L = 12/4.5 \approx 2.67$ square meters per capita. Substituting into equation (8) the per capita income Y=100 thousand RMB, rent $r = 2.78$ thousand RMB, price-income ratio $ratio^{py} = 11.88$ and price-rent ratio $ratio^{pr} = 35.6$ as well as return on housing investment $R = 1.0495$, we obtain $Z = 1.47$.

4.3 House price, Rent and Land Price in the BGP

We have used information from the Hong Kong market such as per-capita income, rent, land use, and price-income and price-rent ratios to identify the parameter Z. Now we use the projected per capita income and land in Beijing to obtain house price and rent when the Beijing market enters the BGP.

From the housing supply equation it is straightforward to obtain

$$\tilde{r}_t = \left(\frac{1}{Z}\right)\left(\frac{ratio^{py}}{ratio^{pr}} \times \frac{Y_t}{L_t}\right)^{\theta}\left[\frac{1-(1-\delta)/R_{t+1}}{1-\theta}\right]^{1-\theta} \tag{9}$$

To calculate the equilibrium rent when the economy enters the BGP, we need to predict Y_{BGP} and L_{BGP}, the income and land supply in Beijing at $t = T_{BGP}$.

Based on the projected evolution of income, land supply and urban population structure, by the time Beijing market reaches the BGP, per capita income is $Y_{BGP} = 871$ thousand in 2014 RMB, and per capita land supply is $L_{BGP} = 12.83$ square meters. Substituting these numbers and other related values into equation (9), we obtain $r_{BGP} = 4.31$ thousand RMB per square meter. This is the market-clearing annual rent for Beijing $t = T_{BGP}$.

House price at the BGP is calculated as r_{BGP} divided by the price-rent ratio. This gives $P_{BGP} = 153.5$ thousand in 2014 RMB per square meter. The corresponding land price at the BGP is 854 thousand RMB per square meter. Thus land price is 5.6 times that of house price at the time the economy enters the BGP.

At the BGP, housing size in Beijing will be $H_{BGP} = 67.36$ square meters per capita based on equation (7). The implied floor-area ratio is $\text{FAR}_{BGP} = H_{BGP}/L_{BGP} = 5.25$. By contrast, in 2014, housing size is about 30 square meters per capita and FAR is below 1.8.

4.4 Household Preference Parameters

To pin down the four parameters related to consumer preference, $(\gamma, \mathcal{B}, \beta, \omega)$, we take a moment-matching approach. Specifically, we pick the set of parameters so that the following six moments generated from the model match as closely as possible those from the data: (i) average price-income ratio; (ii) average price-rent ratio; (iii) home ownership rate; (iv) the average age of first-time home buyers; (v) clearing of the rental market in the BGP; (vi) clearing of the housing equity market in the BGP.

To generate model moments, we simulate for each generation 1,000 households with independent idiosyncratic income and medical expense shocks over their life cycle, compute the optimal decisions of each household in the BGP along each path, then calculate the related moments by taking the average values across the 1,000 simulated households.

For each of the moments used in model calibration, Table 2 shows its target value and the fitted value from our calibrated model. The average age of first-time home buyers contains important information about preference for home ownership as well as parameters related to wealth accumulation, such as β and γ. We could not find data on the average age of first-time home buyers in Hong Kong. But the age at first marriage is about 30 according to the Census and Statistics Department of Hong Kong. The age of first-time home purchase is usually older than the age at first marriage, therefore we use 33 as the age of first-time home purchase. As a further reference, in the US, the average age at first marriage is 28 and the age of first-time home purchase is 34 according to the 2009 American Housing Survey.

The home ownership rate target is 0.75 for the Beijing market in the BGP, higher than the rate of 52% in Hong Kong.[5] According to the 2012 wave of the China Household Finance Survey, average home ownership rate in the first-tier cities in China is over 80%. Based on the history of economies that experienced successful economic transition, we do not expect home ownership rate in Beijing to decline significantly in the future.

Table 2: Moments

Moments	Target	Model
price/income	11.88	11.94
price/rent	35.6	35.6
home ownership rate	0.75	0.82
age of first time buyers	33	32
surplus (consumption MKT)	0	0.0
surplus (investment MKT)	0	0.004

The last column of Table 2 shows the simulated moments. They match the data moments well, except that home ownership rate is somewhat higher in our model. This may arise from our assumption that housing investment is risk-free and costless to adjust, thus homeownership is more attractive in our model relative to the reality.

5 Instability of Empirical Relations

Previous studies tend to estimate the relation between house price and economic fundamentals such as income and land supply with linear regressions (Case and Shiller, 2003). This practice works well in developed market. However, in a transition economy where the evolutions of fundamentals are non-stationary, the relation between house price and fundamentals is unstable and time varying. An attempt to estimate the housing price based on

[5] Home ownership rate in Hong Kong is available from Hong Kong Census and Statistics Department, `http://www.censtatd.gov.hk/hkstat/sub/sp150.jsp?tableID=005&ID=0&productType=8`.

its historical relation with the fundamentals could produce misleading results in a transition economy.

Table 3: Linear regression results

Dependent variable:	$log(G_p)$		$log(G_r)$	
	income	land	income	land
2005-2040	0.710	-0.797	0.670	0.321
2041-2080	1.161	-3.020	0.761	-1.135
2081-2114	0.807	0.140	0.856	-0.916

This table reports the estimated coefficients when we regress log price growth (or log rent growth) on log income growth and log land supply growth using the equilibrium path of price or rent and the exogenous fundamentals, for various subperiods during the economic transition.

To illustrate this point under our calibrated model, we run two regressions using simulated data to contrast the relation between house price and economic fundamentals in the BGP and during the transition periods. First, from points 5-6 in the ***Proposition***, we have the following in the BGP:

$$log(G_p) = \theta log(G_Y) - \theta log(G_L) \tag{10}$$

$$log(G_r) = \theta log(G_Y) - \theta log(G_L) \tag{11}$$

For the Beijing market, when we regress log-price growth on $log(G_Y)$ and $-log(G_L)$ restricting the coefficients to be identical using simulated data in the BGP, we recover a coefficient of 0.72, just as the model predicted (recall that we use a parameter value $\theta = 0.72$ in the calibrated model). The same holds when we use log-rent growth as the dependent variable.

Second, we run another regression using the equilibrium path of price or rent and exogenous fundamentals during various subperiods of the economic transition, but without requiring the coefficients before $log(G_Y)$ and $-log(G_L)$ to be the same. Table 3 reports the results. The difference in the estimated coefficients across various periods is evident. Theoretically, for a transition economy, the relation between house price (or rent) and the fundamentals such as income, land supply and age distribution of population can be non-linear and time varying because of the non-stationarity of the variables. Linear regressions assuming constant parameters would fail to capture these complex dynamic dependences.

6 Additional Robustness Checks

The baseline calibration implies an equilibrium house price in 2014 of 19,410 RMB per square meter that is significantly lower than the 28,194 RMB observed in the data. In Section 5.3, we find that the model equilibrium house price under alternative assumptions with reasonable model parameters are still are substantially lower than the data. Here, we conduct several further robustness check.

6.1 Idiosyncratic Uncertainty

First, we check how our results depend on the amount of idiosyncratic risk faced by the households, as represented by the standard deviations of income and medical expense shocks. This exercise not only helps us quantify the importance of precautionary savings, but also provides insights about potential benefits from policies such as social insurance and medical insurance programs.

The presence of medical expense shocks and income shocks give the households a precautionary motive to save in our model. The precautionary demand for housing equity increases with the idiosyncratic risks and housing consumption decreases with the idiosyncratic risks. Thus, other things equal, a larger idiosyncratic risk would lead to higher equilibrium house price and lower rent in our model.

If the standard deviation of income shocks is cut by 50%, then we observe a lower house price but a higher rent, both in 2014 and 2114, as shown in row (1) of Table 4. This is caused by a reduced precautionary demand for housing asset and the stronger consumption demand due to the lower income risk. Quantitatively, house price is lowered by 3% and rent is raised by 0.6% in 2014 compared with the baseline model. By the time the economy converges to the BGP, house price is lowered by 6% and rent is raised by 1.9%.

Row (2) of Table 4 reports the results when we cut the standard deviation of medical expense shocks by 50%. The effect is qualitatively the same as that of lower income risk, but the magnitude is smaller. In an unreported sensitivity check, we completely remove the idiosyncratic shocks. This causes the current house price to drop by 9.4% and rent to rise by 15.8%. Thus the precautionary savings motive created by idiosyncratic income and medical expense shocks plays a sizable role in generating high house price and price-income ratio in the Beijing market.

6.2 Alternative Reference City

The results reported in the main paper are obtained using Hong Kong, a well-developed housing market, as a good reference for Beijing when it enters the BGP. These two cities have similar cultural background, preferences over housing, and risk tolerance. They both adopt a land lease policy which has been shown to have a significant impact on housing price dynamics (Anglin et al., 2014).[6] One legitimate concern is that the two cities are connected in their economies and differ in the land expansion possibilities. Here, we recompute the equilibrium using Washington, D.C. as an alternative reference city. Unlike Hong Kong, housing market in Washington, D.C. is minimally tied to Beijing. In addition, Washington, D.C. is similar to Beijing in that land is open and amenable to new construction.

In the recalibration, we start with four household preference parameters $(\gamma, \mathcal{B}, \beta, \omega)$. These parameters are pinned down by matching the moments as shown in Table 2, with the price-income ratio and price-rent ratio of 31.1 and 9.08 respectively, calculated as the post-2010 average ratios observed in Washington, D.C. The resulting parameters are γ=2.09, $\mathcal{B}$=16.32, β=0.998, and ω=0.25. Compared with the baseline calibration, a larger γ indicates a smaller EIS while a smaller ω indicates weaker consumption demand for housing.

For housing production parameters, we first pin down the land share parameter θ using equation (6) based on the growth rates of house price and land price in Washington DC.[7] This gives $\theta = 0.60$, which is smaller than $\theta = 0.72$ in the baseline calibration. The lower θ arises from the relatively abundant land supply in Washington, DC which yields a lower growth rate of house price relative to land price compared to Hong Kong.

To pin down the production efficiency parameter Z, we use equation (**??**) which implies $Z = FAR^{\theta} / \left[\frac{(1-\theta)r}{1-(1-\delta)/R}\right]^{1-\theta}$, where FAR, the floor-area ratio in the BGP, is taken to be 3.[8] Using $r = 16.53$ USD per square foot and $R = G_p + (r/p) = 1.05$, we obtain $Z = 0.90$ which is substantially smaller than $Z = 1.47$ in the baseline calibration. The smaller value of Z

[6] Land lease affects house price and rent through the uncertainties associated with the renewal of land lease: renewal is not guaranteed and the cost of renewal is not certain. Such uncertainties not just affect house price directly, but also indirectly through altering the timing and frequency of home redevelopment. It is beyond the scope of our paper to explicitly model the effects of the land lease policy.

[7] Using the 1987-2015 Case-Shiller house price index, the real growth rate of house price is 1.94%. The growth of land price between 1987-2015 is taken from the Lincoln Institute (`http://www.lincolninst.edu/subcenters/land-values/metro-area-land-prices.asp.`).

[8] This number is lower than the present FAR in Hong Kong, but higher than the present FAR in Washington, D.C. Housing in D.C. is mainly stand-alone single-family houses while in Beijing, it is predominately condominiums. FAR=3 in the BGP is a more realistic assumption for Beijing because the FAR of condominiums built in recent years in Beijing already exceeds 2.5.

reflects the lower rise buildings featured in the reference city compared to Hong Kong.

Finally, we infer the growth of land supply (G_L) from the sixth point in the ***Proposition***, $G_p = (G_Y/G_L)^{\theta}$. For Washington, D.C., we obtain $G_Y = 1.056$ and $G_p = 1.020$, which yields $G_L = exp(log(G_Y) - 1/\theta log(G_p)) = 1.02$. This is about the average land supply growth of seven U.S. cities featuring rich land supply: Chicago, Cincinnati, LA, Minneapolis, Philadelphia, Pittsburgh and Tacoma.[9] Note that with 2% of land growth instead of 0.5% as in the baseline specification, Beijing has acquired much more land expansion opportunities when its future path is based on Washington, D.C.

Row (3) of Table 4 reports that under the above full recalibration with Washington, DC as the reference city, Beijing house price in 2014 is 23.69 thousand RMB. Compared to the case of Hong Kong, using Washington, DC to resemble the future of Beijing lowers housing demand (reflected by smaller ω), putting a downward pressure on equilibrium price. However, the supply side effects on house price are mixed. On the one hand, higher land growth (reflected by larger G_L) increases land supply. On the other hand, lower building density and height (reflected by lower θ and Z) effectively reduce the efficiency of land use.[10] It turns out that the production efficiency effect that tightens housing supply dominates the demand effect and the land growth effect, yielding a higher equilibrium price than in the baseline specification. Nevertheless, the model-implied price in 2014 is still 16% below the market price, suggesting that the substantial gap remains between model predicted and observed market prices.

One concern with the above full recalibration is that low urban density and building height in Washington, DC do not present a realistic analogue for the urban configuration of Beijing. To address this, we further present a partial recalibration where housing production parameters remain the same as in the baseline specification but household preference parameters are recalibrated to match the Washington, DC data. The resulting building density and floor-area ratio are closer to what we observe in Beijing. By restricting the supply side as in the baseline calibration, we should expect that the demand side effect will dominate and hence the equilibrium price should be lower compared to the baseline specification. Row (4) of Table 4 shows that the model-implied price is 14.87 thousand RMB in 2014, which is about half of the market price.

[9]See `http://www.lincolninst.edu/subcenters/atlas-urban-expansion/global-sample-cities.aspx`.

[10]With Washington DC as a reference city, FAR is 3 when Beijing enters the BGP as opposed to 5.2 as in the baseline case; housing size per capita is 36.7 square meters in the BGP as opposed to 65 square meters as in the baseline calibration.

Table 4: Price and rent under alternative assumptions (in thousands of 2014 RMB)

		Price		Rent	
		2014	2114	2014	2114
	Data	28.19	n.a.	0.74	n.a.
	Baseline	19.41	153.54	0.56	4.31
(1)	Half idiosyncratic income risk	18.83	144.32	0.57	4.39
(2)	Half medical expense risk	19.37	151.79	0.57	4.31
(3)	Washington, D.C. (full recalibration)	23.69	105.34	0.81	3.39
(4)	Washington, D.C. (partial recalibration)	14.87	122.81	0.53	3.95

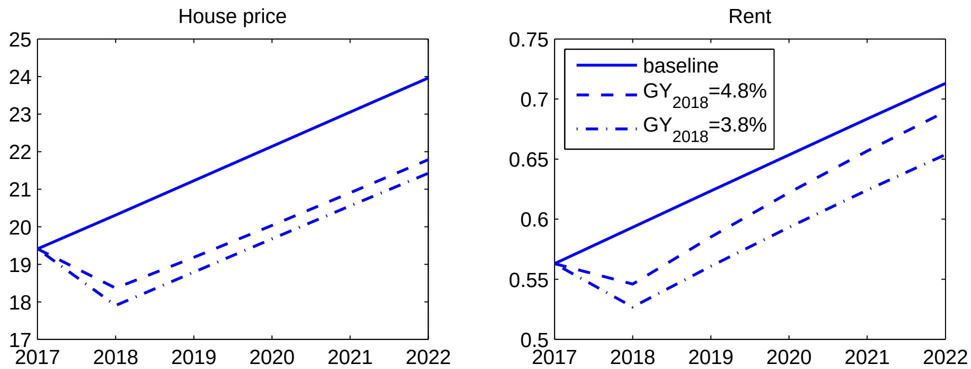
This table reports the equilibrium price and rent for the Beijing housing market in 2014 and 2114 under alternative model assumptions.

6.3 Unexpected Change in Income Growth

Finally, we conduct a robustness exercise regarding the quantitative implications of an unexpected change of income growth. A sudden decline in the growth of disposable household income is one of the major risks facing the current Chinese housing market (Fang et al. (2016)). To capture this possibility, we assume that households make optimal choices based on expectation of future income growth as assumed in the baseline model, until in the beginning of 2018, a sudden drop in income growth is realized, and then they adjust expectation of future income growth downwards and re-optimize accordingly.

Figure 2 plots the paths of price and rent when income growth unexpectedly drops from 5.8% in 2018 (under the baseline scenario) to either 4.8% or 3.8% and subsequently declines linearly to 3% by 2044. In response to an unexpected negative income shock, both house price and rent drops immediately, then grows steadily again over time, albeit from a lower base. The reason is that a sudden decline of income growth brings strong and immediate income effect. The decline of house price when income growth drops from 5.8% to 4.8% is sharp, but the further drop of income growth to 3.8% causes only mild decline in house price. In the case that income growth in 2018 drops to $GY_{2018} = 3.8\%$, house price and rent decline by 11.9% and 11.2% respectively. By the time the economy enters the BGP, house price and rent are 133.45 and 3.76 thousand RMB. Compared with the baseline results, the case of $GY_{2018} = 3.8\%$ lowers house price and rent in the BGP by 13.1% and 12.8% respectively.

Figure 2: House price and rent in response to an unexpected drop in income growth



This figure plots the paths of price and rent when the income growth rate drops suddenly in 2018.

7 A Model with Labor Costs in Housing Production

In the main body of the paper, we have assumed a housing production function that has two inputs: capital and land. Here, we extend the model to include labor as an additional input in the housing production function and examine its impact on the equilibrium outcomes. The new housing production function is

$$H_t = ZB_t^{\zeta} L_t^{\theta} K_t^{1-\zeta-\theta}, \tag{12}$$

where B_t denotes the labor input and $\zeta \in (0,1)$ describes the relative importance of labor.

The firm's optimization problem becomes

$$\max_{K_t, L_t, B_t} \quad \tilde{r}_t H_t - [K_t - (1-\delta)K_{t-1}] - q_t(L_t - L_{t-1}) - w_t B_t + \frac{1}{R_{t+1}} V(K_t, L_t),$$

subject to equation (12). Here w_t is the price of labor (wage of builders) at time t. First order conditions with respect to K_t, L_t and B_t are

$$\begin{aligned} Z(1-\zeta-\theta)\tilde{r}_t B_t^{\zeta} L_t^{\theta} K_t^{-\zeta-\theta} &= 1 - \frac{1-\delta}{R_{t+1}}, & (13) \\ Z\theta \tilde{r}_t B_t^{\zeta} L_t^{\theta-1} K_t^{1-\zeta-\theta} &= q_t - \frac{q_{t+1}}{R_{t+1}}, & (14) \\ Z\zeta \tilde{r}_t B_t^{\zeta-1} L_t^{\theta} K_t^{1-\zeta-\theta} &= w_t. & (15) \end{aligned}$$

From these first order conditions, we derive the following optimal inputs of capital and labor relative to land

$$\begin{aligned} K_t &= \frac{1-\zeta-\theta}{\theta}\frac{q_t - q_{t+1}/R_{t+1}}{1-(1-\delta)/R_{t+1}} L_t, &(16)\\ B_t &= \frac{\zeta}{\theta}\frac{q_t - q_{t+1}/R_{t+1}}{w_t} L_t. &(17)\end{aligned}$$

In the equilibrium analysis we solve B_t from equation (17), taking w_t as exogenously given.[11]

Substituting the above two equations into equation (12), the housing units produced in the extended model is

$$H_t = Z\left(\frac{\zeta}{\theta}\right)^{\zeta}\left(\frac{1-\zeta-\theta}{\theta}\right)^{1-\zeta-\theta}\frac{(q_t - q_{t+1}/R_{t+1})^{1-\theta}}{w_t^{\zeta}[1-(1-\delta)R_{t+1}]^{1-\zeta-\theta}} L_t. \tag{18}$$

As in the baseline model, land price $(q_t - q_{t+1})/R_{t+1}$ in equation (18) can be substituted out. Multiply L_t to both sides of equation (14), we have $\theta r_t H_t = \left(q_t - \frac{q_{t+1}}{R_{t+1}}\right) L_t$, implying

$$q_t - \frac{q_{t+1}}{R_{t+1}} = \frac{\theta \tilde{r}_t H_t}{L_t}. \tag{19}$$

Substituting (19) into (18), we obtain the following expression for the housing supply:

$$\begin{aligned} H_t &= Z^{1/\theta}\zeta^{\zeta/\theta}(1-\zeta-\theta)^{(1-\zeta-\theta)/\theta}\left[\frac{\tilde{r}_t^{1-\theta}}{w_t^{\zeta}\left(1-\frac{1-\delta}{R_{t+1}}\right)^{1-\zeta-\theta}}\right]^{1/\theta} L_t \\ &= \left(\frac{Z}{w_{BGP}^{\zeta}}\right)^{1/\theta}\zeta^{\zeta/\theta}(1-\zeta-\theta)^{(1-\zeta-\theta)/\theta}\left[\left(\frac{w_{BGP}}{w_t}\right)^{\zeta}\frac{\tilde{r}_t^{1-\theta}}{\left(1-\frac{1-\delta}{R_{t+1}}\right)^{1-\zeta-\theta}}\right]^{1/\theta} L_t \quad (20)\end{aligned}$$

where w_{BGP} is the wage when the economy enters the BGP. Note that when $\zeta = 0$, equation (20) coincides with the housing supply function in the baseline model.

Using the same strategy as in the baseline model, one can show that a BGP exists. Let G_w be the growth factor of wage, and G_B be the growth factor of labor inputs in the BGP in the extended model. We assume that wage grows at the same rate as per capita income, i.e., $G_w = G_Y/G_N$. Consistent with this assumption in the BGP, we take wage w_t to be proportional to the per capita income during the transition period.[12] In our quantitative

[11] In a full-blown general equilibrium with two production sectors – housing and the numeraire, the wage can be determined endogenously.

[12] We interpret B_t as labor input. More broadly it can also represent other inputs of housing construction such as construction materials. Thus, w_t, the price of B_t, can capture other construction costs such as the cost of material. In general, w_t may have exogenous shocks not tied to the income process and may exhibit some cyclical movements. Although our model and its solution strategy allow such cyclical movements, we focus on the long run trend, which is consistent with the main theme of the paper.

analysis for the Beijing market, $G_w > 1$ in the BGP and wage also rises over time during the economic transition period.

It is straightforward to verify the following growth factors in the BGP in the extended model:

$$\begin{aligned} G_B &= G_N, \\ G_H &= G_w^{-\zeta} G_L^{\theta} G_Y^{1-\theta}, \\ G_p &= G_w^{\zeta} \left(\frac{G_Y}{G_L}\right)^{\theta}, \\ G_{FAR} &= G_w^{-\zeta} \left(\frac{G_Y}{G_L}\right)^{1-\theta}. \end{aligned}$$

The growth factors of other variables are identical to those in the baseline model. In particular, the growth of land price is still $G_q = G_Y/G_L$. Substituting this into the G_p function yields $G_p = (G_Y/G_N)^{\zeta} G_q^{\theta}$, equivalently,

$$log(G_p) = \zeta log(G_Y/G_N) + \theta log(G_q), \tag{21}$$

i.e., the growth rate of house price increases with the growth of land price and the growth of income per capita.

For quantitative analysis, we consider three scenarios for the relative importance of labor: $\zeta = 0.05$, $\zeta = 0.1$ and $\zeta = 0.2$. Since the housing production function is changed due to the inclusion of labor costs, θ and Z need to be re-calibrated. To pin down θ, we use equation (21) and take $G_Y/G_N = 1.015$ based on a 1.5% annual income growth rate in Hong Kong. Table 5 reports these re-calculated values of θ. With the introduction of labor input in the housing production, the relative importance of land in the extended model diminishes compared to the baseline model. Thus, the values of θ under the extended model with labor are lower than the value of θ in the baseline model. As the labor share in the production (ζ) becomes larger, the land share falls and θ becomes smaller.

We pick the production scaling parameter Z so that housing supply in period $t = T_{BGP}$ is the same as that in the baseline model, i.e., $H_{BGP} = 67.36$. This facilitates the comparison between the baseline model and the extended model. By keeping housing supply at T_{BGP} unchanged, the demand side does not change either in the equilibrium, thus there is no need to re-calibrate the household preference parameters. We can use the same household preference parameters of the baseline model which are chosen so that housing demand equals supply at T_{BGP}. Consequently the differences in the equilibrium prices between the extended model and the baseline model are entirely due to the effect of labor on the supply side of the economy.

Specifically we pin down the value of Z using the following equation:

$$\frac{Z}{w_{BGP}^{\zeta}} = \left(\frac{1}{\tilde{r}}\right)\left(\frac{ratio^{py}}{ratio^{pr}} \times \frac{Y}{L}\right)^{\theta}\left[\frac{1-(1-\delta)/R}{1-\zeta-\theta}\right]^{1-\zeta-\theta}\left(\frac{1}{\zeta}\right)^{\zeta}. \tag{22}$$

The special case of equation (22) when $\zeta = 0$ is identical to equation (33) in the baseline model. Clearly Z and w_{BGP} are not identified separately. What matters for housing supply is not Z itself, but the value of Z/w_{BGP}^{ζ} (see equation (20)). Without loss of generality, we normalize $w_{BGP} = 1$ and obtain the value for Z from equation (22), setting Y and L to the projected aggregate income and land supply in Beijing at $t = T_{BGP}$, i.e., $Y_{BGP} = 870.7$ and L_{BGP}= 12.83. The resulting values for Z are reported in the second column of Table 5.

Comparing the growth factor of housing supply (G_H) in the extended model to that in the model without labor costs (see point #2 of Proposition in Section 3.4), we obtain

$$G_H/G_H^{'} = G_w^{-\zeta}\left(\frac{G_Y}{G_L}\right)^{\theta'-\theta} \tag{23}$$

where $G_H^{'}$ and $\theta^{'}$ correspond to the basline model. The two terms on the right hand side of equation(23) correspond to two forces driving the comparison of the growth factor of housing supply in the BGP. The first term $G_w^{-\zeta}$ reflects the effect of rising labor costs in Beijing and it lowers G_H. On the other hand, the second term on the right side of (23) raises G_H, since $\theta < \theta^{'}$ and $G_Y/G_L > 1$ in our quantitative exercises for the Beijing market (i.e., per capita income growth outpaces per capita land supply growth). The second term reduces the constraint of land on housing supply in the extended model because land now has a smaller share in housing production compared to the baseline model. Depending on the parameters, the growth factor of housing supply in the extended model can be either higher or lower than that in the baseline model. In our quantitative analysis, the wage growth in the BGP and ζ are both sufficiently small so that the second effect dominates, leading to a higher growth of housing supply in the extended model (i.e., $G_H > G_H'$).

Similar arguments can explain why the growth factor of housing price in the BGP (G_p) under the extended model becomes smaller in our numerical exercises, as can be seen from growth factors of house price reported in the third column of Table 5. The lower price growth is consistent with a higher growth of housing supply. The key force driving both results is that the limited land supply has become less constraining in the extended model.

A comparison of the growth factor of FAR is also informative. If we incorporate the labor input into the model, the firm will find it optimal to substitute more labor for land in housing production when the growth rate of wage is low, yielding a higher FAR. This implies a higher structural density and taller buildings in the extended model.

Table 5: Effect of labor costs

	θ	Z	G_p	house price (2014)	rent (2014)	land price (2014)	land price (2114)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
baseline	0.719	1.470	1.0214	19.41	0.56	47.14	853.92
ζ=0.05	0.686	1.615	1.0212	18.75	0.54	45.62	824.08
ζ=0.10	0.653	1.638	1.0209	18.16	0.51	43.75	793.60
ζ=0.15	0.619	1.618	1.0207	17.59	0.49	41.86	762.35

This table reports the parameter values related to housing production and the corresponding house price, rent and land price in the extended model with labor costs in housing production.

The table above shows that equilibrium housing price, rent and land price in 2014 all decrease with ζ in the calibrated model with labor costs for the Beijing market. Recall that in the calibration, we have chosen Z to keep the housing supply at T_{BGP} (when the economy goes into the BGP) the same as the calibrated baseline model. This implies that house price and rent are also unchanged at T_{BGP}. However, at $t < T_{BGP}$, housing supply decreases with w_t/w_{BGP} (see equation (20)). Since wage raises over time in our quantitative analysis for the Beijing market, more houses will be built earlier on (when wage is relatively low) and future growth of housing supply becomes lower. This leads to a lower initial housing price, but faster future appreciation of house price.

To summarize, we have analyzed both qualitatively and quantitatively the effects of labor costs on equilibrium outcomes in our model. Qualitatively, the introduction of labor costs lower the house price, rent and land price in the earlier years of the transition period when labor costs are still low but rising quickly. This result is in stark contrast to the naive intuition that housing price would be higher after taking into account the rising labor costs. That intuition is based on partial equilibrium thinking and ignores the general equilibrium effect of labor costs on housing supply. Quantitatively, the equilibrium level and growth in price and rent in our model are not very sensitive to the inclusion of labor costs in the housing production function, especially for cities such as Beijing where limited land supply plays a much larger role than the labor costs (i.e., cities with low ζ and high θ).

References

Anglin, Paul M., David Dale-Johnson, Yanmin Gao, and Guozhong Zhu, "Patterns of Growth in Chinese Cities: Implications of the Land Lease," *Journal of Urban*

Economics, 2014, *83* (C), 87–107.

Case, Karl E. and Robert J. Shiller, "Is There a Bubble in the Housing Market?," *Brookings Papers on Economic Activity*, 2003, *34* (2), 299–362.

Fang, Hanming, Quanlin Gu, Wei Xiong, and Li-An Zhou, "Demystifying the Chinese Housing Boom," *NBER Macroeconomics Annual*, 2016, *30* (1), 105–166.

Henderson, Vernon, "Cities and Development," *Journal of Regional Science*, 2010, *50* (1), 515–540.

Krusell, Per and Anthony A. Smith, "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy*, 1998, *106*, 868–896.

Leigh, Wilhelmina A., "Economic Depreciation of the Residential Housing Stock of the United States, 1950-1970," *The Review of Economics and Statistics*, 1980, *62* (2), 200–206.