To Own or to Rent?
The Effects of Transaction Taxes on Housing Markets*

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Abstract

Using transaction records on housing sales and leases, we estimate the effect of Toronto's imposition of a land transfer tax in 2008. We find two novel effects of an increase in land transfer tax: (1) a rise in buy-to-let transactions but a fall in owner-occupier transactions despite the tax applying to both, (2) a simultaneous fall in the price-to-rent ratio and the sales-to-leases ratio. We develop a housing search model with both ownership and rental markets to show that the land transfer tax can generate lock-in effects both within the ownership market and across the two markets. It accounts for the two new facts by predicting a reduction in mobility within the ownership market and an increase in demand in the rental market which generates a fall in the homeownership rate. The implied deadweight loss as a percentage of tax revenue raised is large at 45%, with half due to distortions within the ownership market and the remaining half due to distortions across the two markets.

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KEYWORDS: rental market, buy-to-let investors, homeownership rate, land transfer tax.

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1 Introduction

Transaction taxes have a long history that goes back to the 17th century to raise money for war. Over time, transaction taxes on stocks and other assets have been scrapped. However, transaction taxes on properties — also called land transfer taxes in North America and stamp duties in Europe — remain a prominent source of tax revenue in over twenty-seven OECD countries today.[1] Across these countries, it has been well documented that land transfer taxes cause large distortions to house prices, transaction volumes, and residential mobility for existing homeowners.

While the prior literature has almost exclusively focused on the distortionary effects within the owner-occupied market, little is known about how the tax distorts households' tenure choice and hence housing allocation across the owner-occupied and rental markets. The latter has first-order importance for policy because increasing homeownership has been a target in many countries. In this paper, our objective is to gain a comprehensive understanding of the effects of land transfer taxes on housing markets, along both the intensive margin, namely moving and transaction decisions, and the extensive margin, namely the decision of owing versus renting.

We use a unique dataset on housing sales transactions and leasing transactions from the Multiple Listing Service (MLS) transaction records in the Greater Toronto Area (GTA). The data allow us to identify property purchases made by buy-to-let investors as opposed to owner occupiers. We estimate the effect of the imposition of a land transfer tax in the City of Toronto in 2008 by comparing the change in a rich array of housing market outcomes across treated and untreated neighbourhoods, based on a regression discontinuity design.

We start by documenting a set of new empirical facts. First, an increase in the land transfer tax (LTT) causes a 10% decline in home purchases but a 9% increase in buy-to-let investor purchases. The opposite effects of the LTT on investors and home buyers are quite striking given that the same tax applies to both groups of buyers.

Second, the LTT causes a 0.7% decline in the fraction of home purchases out of all purchases and a 3.7 percentage point increase in the leases to sales ratio. Both of which are consistent with a recently reversed trend in the homeownership rate,[2] suggesting the important role that the land transfer tax plays in shaping households' tenure choices.

Third, the LTT has a positive effect on the rental yield (the inverse of the price-rent ratio). This is not surprising since the increase in transaction cost should be capitalized into a decline in price. Interestingly, the price paid by homeowners declines more than the increase in the tax, while the

[1] Including the U.K. (Besley, Meads and Surico, 2014, Hilber and Lyytikäinen, 2017, Best and Kleven, 2018), U.S. (Benjamin, Coulson and Yang, 1993, Slemrod, Weber and Shan, 2017, Kopczuk and Munroe, 2015), Canada (Dachis, Duranton and Turner, 2012), Australia (Davidoff and Leigh, 2013), Germany (Fritzsche and Vandrei, 2019), Finland (Eerola, Harjunen, Lyytikäinen and Saarimaa, 2019, Määttänen and Terviö, 2020), and the Netherlands (Van Ommeren and Van Leuvensteijn, 2005).

[2] The homeownership rate, measured by the fraction of properties lived in by their owners at a five-year frequency, had been steadily increasing until 2011 and then decreased from 54.5% to 52.3% after 2011 in the city of Toronto, according to Statistics Canada.

price paid by investors is much less affected and the rents even increase slightly.

Together, these facts reveal novel heterogeneous treatment effects of the LTT on the owner-occupied market versus the rental market. Intuitively, owner-occupiers expect to be subject to the LTT not only once, but each time they desire to move, both directly as buyers of subsequent properties, and indirectly as sellers hit by lower prices. As a result, prices change more than the LTT. With existing homeowners staying in their current property longer and properties taking longer to sell, overall home-buyer transactions decline.

In contrast, by choosing to rent, individuals save themselves from the burden of paying the increased LTT. Thus, the burden of the LTT falls more lightly on the rental market compared to the market for owner-occupied properties. Moreover, renting properties mitigate the impact of the tax on overall house transactions because when tenants desire to move, the landlord does not need to sell and pay the LTT again. In sum, the imposition of the LTT makes owning a home less attractive relative to renting. The increase in demand for rental properties further encourages the entry of buy-to-let investors, which shifts properties from the owner-occupied sector to the rental sector, resulting in a fall in the homeownership rate.

To model explicitly the economic forces we document and quantify the relative importance of the intensive-margin and extensive-margin changes in welfare, we develop and calibrate a model suitable to analyse the housing ownership and rental markets jointly. A crucial feature is that individuals can choose which market to participate in, subject to paying the cost of accessing credit to become a homeowner. These credit costs are heterogeneous across individuals. This gives rise to an entry decision on the buy side of the rental market. On the sell side, there is free entry of buy-to-let investors.[3] The equilibrium homeownership rate is the one consistent with the behaviour of both households and landlords.

Both ownership and rental markets are subject to search frictions where market tightness (the ratio of buyers to sellers) affects the probability of meeting, and idiosyncratic match quality affects the probability of transacting conditional on meeting. Match quality is a persistent variable subject to occasional idiosyncratic shocks, after which homeowners make a decision whether to move, doing so if match quality is below a moving threshold.

An increase in LTT in the model makes existing homeowners more tolerant of poor match quality, and thus reduces moving rates, lengthens holding periods, and reduces housing transactions. The higher LTT also makes it less attractive to be an owner-occupier relative to a renter, so some marginal homebuyers are dissuaded from paying the credit cost to enter the ownership market. Since they must still live somewhere, this raises demand in the rental market and attracts more buy-to-let investors.

The model spells out two facets of the welfare cost of the LTT. First, the ‘lock-in’ effect of less moving within the ownership market. This gives rise to misallocation of properties among homeowners in that average match quality declines. Second, there is misallocation across the markets in

[3] An investor is anyone who owns a property but does not use a property to live in. Thus, an investor simply represents funds invested in housing; the identities of the investors do not matter.

that fewer people will pay the credit cost to access better match quality in the ownership market.

We calibrate the model to the city of Toronto housing market, which has a low homeownership rate of about 54% and an active market for buy-to-let.[4] In February 2008, the effective LTT rate increased by 0.96 percentage points. By matching the change in transactions, we capture the decline in mobility within the ownership market. An average homeowner now lives in a property for 8.4% longer. The price-rent ratio declines by 1.1%, and the time on the market goes up by 6.8%. The decrease in price is 1.4% which is larger than the 0.96% tax change itself, which follows from the anticipation effect of paying again with subsequent moves. The homeownership rate declines by 1.6%, and the number of leases goes up by 1.9%.

The welfare costs of the LTT are substantial. Per dollar of tax revenue raised, the LTT generates a welfare loss equivalent to 45% of the increase in tax revenue. This welfare loss is significantly larger than the 12.5% found by Dachis, Duranton and Turner (2012) for the Toronto LTT. The welfare loss is due to distortions within and across the two housing markets, with each accounting for about half of the loss. We find the large welfare losses are largely due to considering mobility within the ownership market and across the two markets. The fall in mobility within the ownership market implies a fall in the average match quality whereas the fall in rent-to-own flow implies a fall in homeownership rate.

Related literature Stamp duty, which used to require an actual stamp, dates back to the 17th century on various transactions to raise money for war. Over time, transaction taxes on stocks and shares and other goods have been scrapped yet taxes on the transfer of properties are generating increasing revenue (Scanlon, Whitehead and Blanc, 2017, Lenoel, Matsu and Naisbitt, 2018). For example, in the U.K., tax revenue raised from stamp duty land tax has increased eightfold during the two decades from 1997 to 2017 reaching more than 8 billion pounds.[5]

During the last two decades, concerns about the economic costs of stamp duty have grown among policymakers and in academic research. Two prominent ones are the ‘Henry Review’ appointed by the Australian government and the ‘Mirrlees Review’ by the U.K. government. Both reviews found significant costs associated with stamp duties owing to reduced mobility and the distortions associated with ad valorem taxes. The reviews proposed reforms to replace stamp duty with a land value tax or tax on housing consumption (Henry, Harmer, Piggott, Ridout and Smith, 2009, Mirrlees, Adam, Besley, Blundell, Bond, Chote, Gammie, Johnson, Myles and Poterba, 2010).

These findings are confirmed by economists working on housing using data from Australia, Canada, Finland, Germany, the U.S. and the U.K. The majority of the literature has focused on the

[4]Some have argued that rental and ownership markets are distinct segments of the housing market using U.S. national data (Glaeser and Gyourko, 2007, Bachmann and Cooper, 2014). However, when focusing on a market like the City of Toronto, these two segments are more interrelated.

[5]Taxes on property including property tax and land transfer tax are important sources of tax revenue in the OECD countries. The U.K, U.S., and Canada are among the highest with taxes on property amounting to 12–14% of tax revenue. This is substantial considering tax on personal income is about 30–40% in those three countries.

effects on mobility, transaction volumes, or house prices.[6] Among them, a few have also computed the welfare cost of public funds per unit of tax revenue: Dachis, Duranton and Turner (2012) for Toronto, Hilber and Lyytikäinen (2017) and Best and Kleven (2018) for the U.K., Eerola, Harjunen, Lyytikäinen and Saarimaa (2019) and Määttänen and Terviö (2020) for Finland, and Fritzsche and Vandrei (2019) for Germany. These losses are associated with the intensive margin through the reduction in transactions and mobility within the ownership market. But as Poterba (1992) wrote many years ago, "*finding the ultimate behavioral effects requires careful study of how tax parameters affect each household's decision of whether to rent or own as well as the decision of how much housing to consume conditional on tenure.*"

We make two contributions to this literature. First, empirically we document the different response of buy-to-let investors and the effects on the price-to-rent ratio, which show the importance of the extensive margin. Second, we develop and quantify a housing search model featuring search in both ownership and rental markets with an endogenous moving decision within and across the two markets.

There is a large body of literature (starting from Wheaton, 1990, and followed by many others) that studies frictions in the housing market using a search-and-matching model as done here. That extensive literature is surveyed by Han and Strange (2015).[7] Among those, Lundborg and Skedinger (1999) explicitly study the effect of transaction taxes on search effort in a version of the Wheaton (1990) model. Since they abstract from the rental market and the decision to move, their model cannot be used to study the impact of transaction taxes on homeownership and mobility.

While the majority of housing search models have abstracted from search in the rental market, recent papers by Halket, Pignatti and di Custoza (2015) and Ioannides and Zabel (2017) have explicitly taken into account search in both ownership and rental markets.[8] While their objectives are different from ours, with the former focusing on the relationship between rent-to-price ratios and homeownership across sub-markets, and the latter focusing on the Beveridge curve in the housing market, they abstract from the moving decision that is crucial in our setting in driving effects on both extensive and intensive margins.

The application of our theory is close to Dachis, Duranton and Turner (2012) in studying the effects of the 2008 Land Transfer Tax (LTT) in Toronto. We combine the transaction-level sales data with the rental data, both from the Multiple Listing Service transaction records. We examine

[6]See, for example, Lundborg and Skedinger (1999), Van Ommeren and Van Leuvensteijn (2005), Davidoff and Leigh (2013), Besley, Meads and Surico (2014), Dachis, Duranton and Turner (2012), Kopczuk and Munroe (2015), Hilber and Lyytikäinen (2017), Best and Kleven (2018), Eerola, Harjunen, Lyytikäinen and Saarimaa (2019), Fritzsche and Vandrei (2019), and Määttänen and Terviö (2020).

[7]For recent examples, see Anenberg and Bayer (2020), Díaz and Jerez (2013), Gabrovski and Ortego-Marti (2019), Guren (2018) Head, Lloyd-Ellis and Sun (2014), Moen, Nenov and Sniekers (2021), Ngai and Tenreyro (2014), Ngai and Sheedy (2020), Piazzesi, Schneider and Stroebel (2020), and Genesove and Han (2012).

[8]There is also a literature on understanding changes in the homeownership rate using models without search, for example, Chambers, Garriga and Schlagenhauf (2009), Fisher and Gervais (2011), Sommer and Sullivan (2018), and Floetotto, Kirker and Stroebel (2016). See Goodman and Mayer (2018) for a survey of the determinants of homeownership.

an array of market outcomes above and beyond sales prices and volumes, which allows us to gain a comprehensive understanding of how the housing market reacts to the LTT. Using a partial equilibrium model for the ownership market, Dachis, Duranton and Turner (2012) computed the welfare loss of the LTT to be 1/8 of tax revenue. By considering a general-equilibrium search model with endogenous moving within and across the ownership and rental markets, we find a much larger loss of around 45% of tax revenue.

Two recent paper with a related objective to our are Kaas, Kocharkov, Preugschat and Siassi (2021) and Cho, Li and Uren (2021) in analyzing the effect of stamp duty on the homeownership rate and its implications for welfare in a model without search frictions. A key advantage of our paper is that we identify the differential effect of stamp duty on buy-to-let investors and owner-occupiers using data on leasing and transaction records. On the theory side, we highlight the indivisible nature of housing and separately allow for buy-to-let investors in a search model to capture the differential effect we find empirically.

2 Data and stylized facts

Our empirical analysis uses data from the Greater Toronto Area to study the effect of an increase in Land Transfer Tax in the City of Toronto in 2008. After February 2008, the new Land Transfer Tax (LTT) affected all real estate transactions occurring in the city of Toronto, while the rest of the Greater Toronto Area had the same provincial-level LTT as before. See Appendix Table 7 for the LTT tax rates in Toronto and the Province of Ontario.

2.1 Data

Our data for housing sales transactions and leasing transactions come from the Multiple Listing Service (MLS) transaction records in the Greater Toronto Area (GTA), the fourth largest North American Metropolitan area. The data cover residential property transactions from 2000 to 2018 and lease transactions for 2006–2018. For each sale transaction, we observe house price, time on the market, transaction date, the exact address and neighbourhood. For each lease transaction, we observe listing date, lease date, monthly rent and lease term, and the exact address and neighbourhood. For all sale or lease transactions that occurred after 2006, we also observe detailed house characteristics such as the number of bedrooms, the number of washrooms, the number of kitchens, lot size (except for condominiums/apartments), house style, family room style, basement structure/style, and heating types/sources.

Using these rich data, we examine housing market outcomes both at the market segment level and at the transaction level. At the market level, housing outcomes include the number of sales, sales to homeowners, sales to investors, the number of leases, leases-sales ratio, price-rent ratio, average price paid by homebuyers and by investors, average rent, average time on the market for sellers and for landlords and average lease term, all measured at the market segment level. A market segment is defined by property type $\times$ community $\times$ year $\times$ month. A community refers to a neighbourhood.[9] Property types include houses, townhouses, condominiums, and apartments. At the transaction level, housing outcomes include sales price, time on the market for sellers and the length that a homeowner stays in her current home.

Since we observe detailed address and transaction dates, we can identify the properties that appear in both datasets by property address. Doing so allows us to generate two novel measures that link the rental and owner-occupied sectors in the housing market. First, if a sale of a property is followed by being listed on the rental market between 0 and 18 months after the sale, we identify it as a buy-to-rent transaction. Alternatively, if a sale of property is followed by being listed for sale between 0 and 18 months after the original sale, we identify it as a buy-to-sell transaction.[10] The remaining sales transactions are considered as home purchase transactions. Second, for buy-to-rent transactions, we impute the property-level price-rent ratio. This is different from the average price-rent ratio that is often used to measure housing market conditions (Shiller, 2007). This ratio ignores the fact that properties underlying average rent and properties underlying average sales price are often not comparable. For instance, Glaeser and Gyourko (2007) use the 2005 American Housing Survey and show that "the median owner occupied unit is nearly double the size of the median rented housing unit", and that rental units are more likely to be located near the city centre. Following Bracke (2015), we address this concern by deriving the price-rent ratio from the sales price and rent for the same property that was leased out within a year after a sale.[11]

2.2 Stylized facts

In this section we present stylized facts about the Greater Toronto Area (GTA) housing market and preliminary evidence linking the 2008 Land Transfer Tax with housing market outcomes.

The Greater Toronto Area has experienced a persistent housing boom since 2000. There was a substantial increase in the homeownership rate from 63% to 68% during 1996–2006 in the GTA, followed by no growth until 2011 and a 2 percent decline between 2011 and 2016. As shown in Figure 1, the City of Toronto follows the same pattern with an increase from 51% to 54% during 1996–2006, followed by almost no growth and then a decline to 53% in 2016.

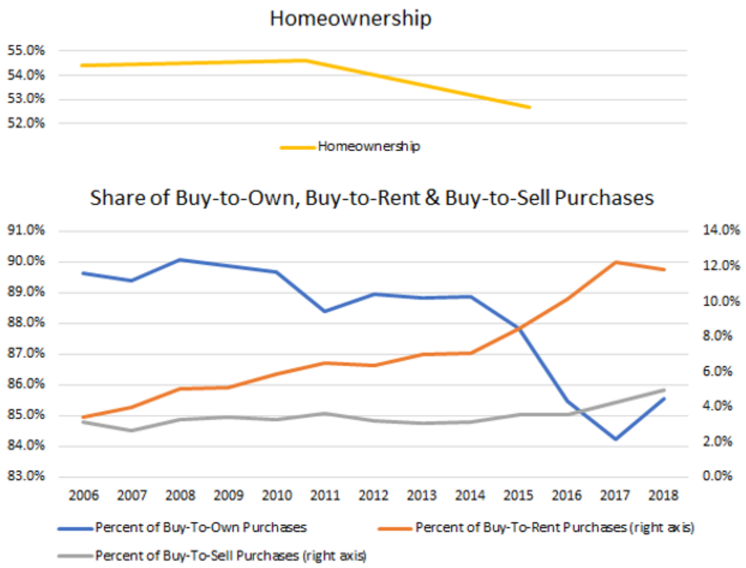
In the background of the stagnation in the homeownership rate was a dramatic increase in buy-to-let purchases. As shown in Figure 1, among all residential home transactions by individual buyers in the City of Toronto between 2006-2017, the fraction of buy-to-own transactions declines from 89%

[9] There are 296 communities in the Greater Toronto Area, including 140 communities in the city of Toronto. See https://www.toronto.ca/city-government/data-research-maps/neighbourhoods-communities/neighbourhood-profiles/.

[10] For robustness check, we change the 18 month threshold to 6, 12, and 24 months, respectively. The estimation results do not change significantly.

[11] Using a similar approach, Rutherford, Rutherford and Yavas (2021) impute the price-rent ratio using the matched sales and rental data in Miami-Dade County.

Figure 1: *Declining Owners and Increasing Investors*



to 84% while the fraction of buy-to-rent transactions triples from 4% to 12%. In contrast, the fraction of buy-to-sell transactions remains stable at around 4% throughout most of the sample period. For this reason, we abstract from buy-to-sell transactions in this paper.

While there are many factors that could affect the inflow of buy-to-rent investors,[12] we restrict our focus to a natural experiment arising from the introduction of Toronto's Land Transfer Tax (LTT) on real estate purchases in February 2008.[13] After February 2008, the LTT affected all real estate transactions occurring in the city of Toronto, while the rest of the Greater Toronto Area had the same provincial-level LTT as before. Appendix Table 7 summarizes the LTT schedule for the city of Toronto before and after February 2008. The effective LTT rate, measured by the mean of land transfer taxes relative to home sales prices across transactions, is 1.08% during the pre-policy period and 2.04% during the post-policy period for the city of Toronto. This implies a 0.96 percentage points increase in the effective LTT.

Figures 2–4 illustrate the relationship between the LTT and changes in housing market outcomes between the two 12-month periods before and after February 2008. In each figure, the line describes the change in the market outcome in an average community in the post-policy period as a percentage of average price in the pre-LTT period, as the distance from the Toronto border varies. Three patterns emerge. First, as shown in Figure 2, house price growth is a smooth function of the distance to the CBD with a discrete downward jump right at the Toronto city border, confirming the price capitalization effect of the LTT in the existing literature. Second, Figure 3 indicates a downward discrete

[12]For example, in June 2021, Blackstone Group Inc announced to buy a company that buys and rents single-family-homes in a $6 billion deal. See https://www.wsj.com/articles/blackstone-bets-6-billion-on-buying-and-renting-homes-11624359600?st=15p9uzchq3djfe4reflink=article$_i$*message$_s$hare*.

[13]Table 7 in the appendix lays out the details about the LTT policy.

Figure 2: *Average sales prices across the Toronto border*

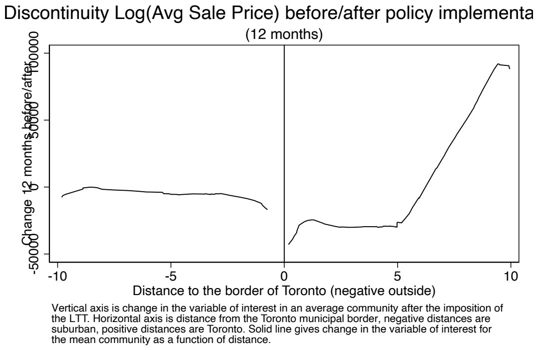


Figure 3: *Buy-to-Own Transactions Across the Toronto border*

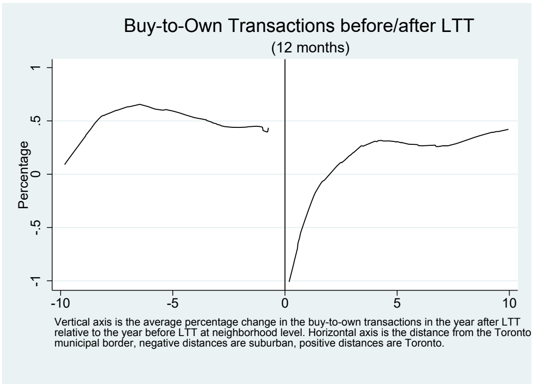
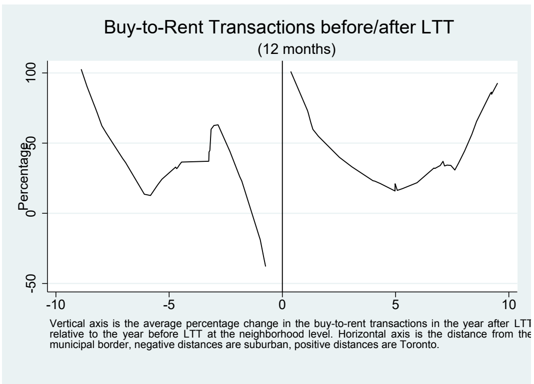


Figure 4: *Buy-to-Rent Transactions Across the Toronto border*



jump at the city border in the number of buy-to-own transactions in the years after the LTT relative to the years before the LTT, reflecting the standard lock-in effect of the LTT. Strikingly, this is not the case for buy-to-rent investors. Instead, Figure 4 indicates an upward discrete jump at the city border in the number of buy-to-rent transactions after the LTT, suggesting there is a reallocation of housing stock from the owner-occupied to the rental sector. The last two sets of stylized facts reveal a novel heterogeneous treatment effect of the LTT on the rental market versus owner-occupied homes.

While the evidence here is illustrative, its interpretation faces two challenges. First, it does not reveal the mechanism driving the differential treatment of the LTT on homebuyers and buy-to-rent investors. Second, it does not isolate the effects of the policy on housing markets from other potential confounding factors. This is particularly concerning because the implementation of the LTT coincided with the initial stage of the 2008 financial crisis, which may have affected the city and suburban areas differently. To address the first challenge, we leverage the richness of our transaction-level sales and lease transaction records. We compare the change in a rich array of housing market outcomes across treated and untreated neighbourhoods and interpret the patterns through the lens of a search model featured both ownership and rental markets. To address the second issue, we rely on a regression discontinuity design and focus on hedonically similar homes in relatively homogenous neighborhoods adjacent to the city border. As indicated in Figures 2–4, the LTT caused two discrete changes in the housing market: one at the border of the city of Toronto, the other on the date the LTT was imposed. This gives opportunity for us to establish the causal effects of the LTT on the housing market.

3 Estimating the effects of the land transfer tax

Our main empirical specifications resemble closely to the regression discontinuity design in Dachis, Duranton and Turner (2012). We estimate the effects of the LTT by comparing the change in housing market outcomes for neighbourhoods in Toronto that are 'treated' with the LTT to untreated suburban neighbourhoods. While Dachis, Duranton and Turner (2012) focus entirely on the short-run (six month) effects of the LTT on transaction volume and sales price in the single family housing market, we depart from their analysis in three ways.

First, we extend the sample to cover not only a longer time period (two years before and after the policy) but also different property segments including single family houses, townhouses, condominiums and apartments. Second, we combine the transaction-level sales data with the rental data, both from the Multiple Listing Service transaction records. Since the LTT only applies to property purchases, the comparison allows us to filter out macro trends and local economic shocks that affect housing market outcomes in general. Third, we examine an array of market outcomes above and beyond sales price and volume, which allows us to gain a comprehensive understanding of how the housing market reacts to the LTT.

Specifically, we restrict the sample to the two years before and two years after February 2008: we define January 2006 – January 2008 as the pre-policy period and February 2008 – February 2010 as the post-policy period. We regress a set of housing market outcome variables on the interaction of a dummy indicating the city of Toronto and a dummy indicating the post-policy period. Given that the LTT was implemented for the city of Toronto in February 2008, we rely on the coefficient on the interaction of the dummies to capture the possible LTT effects. To ensure relatively homogeneous housing stock and neighbourhoods in our sample, we control for house characteristics and compare properties on the opposite sides of the city border — the geographic lines that determine whether the LTT is applicable. By limiting the sample to properties in close proximity to each other but on opposite sides of the borderline, we control for neighbourhood and housing stock differences. Importantly, the possibility that housing market outcome variables make a discrete jump at the border while neighbourhoods continue to change in a smooth manner allows us to isolate the relationship between the LTT and housing market outcomes.

The underlying assumption is that housing market outcomes in treated and untreated neighbourhoods experienced similar housing market trends in the absence of the LTT. The validity of our strategy rests on three assumptions. First, the real estate market did not anticipate the tax. Second, no other policy changes differentially affected the Toronto and suburban real estate market at the same time when the LTT was imposed. Third, there is no substantial sorting of buyers from inside to outside the border in response to the LTT. We relax these assumptions in robustness checks.

Table 1 presents results from the market-segment level regressions where a unit of observation is a market segment defined by property type × community × year × month. Table 2 presents results from the transaction level regressions where a unit of observation is an individual transaction. All regressions are estimated with community fixed effects, year fixed effects, month fixed effects, property type fixed effects, and their interactions. This allows us to control flexibly for the differential evolution of housing market outcomes across different property types in different communities. Table 2 further controls for a rich set of time-varying house characteristics at the transaction level.

3.1 Transaction volume and homeownership

We start with the LTT effects on transaction volume. Row 1 of Table 1 examines the LTT effects on total sales volume. Column (1) indicates that the implementation of the LTT is associated with a 9.8% decrease in sales. In column (2), we include 6 monthly indicators (3 months leading and 3 months following the implementation of the LTT) for neighbourhoods in Toronto to condition out the run-up in sales volume right before the policy and possible anticipation effects. The estimated LTT effect is reduced to 7.5%. In column (3), we further add a linear time trend for neighbourhoods in Toronto, allowing for the possibility of spatially differentiated housing market trends in and outside of Toronto. The estimated LTT effect changes slightly to 7.9%, indicating that the Toronto-specific time trend is not affecting our estimates much. Given that this specification has most extensive controls, we retain it as the main specification from now on.

In column (4), we replicate our preferred regression of column (3) but extend our sample of observations to consider all properties sold within five kilometres of the Toronto border instead of three. In column (5), we further explore the possibility of a spatially differentiated effect of the LTT by adding a coefficient for Toronto properties between two and a half and five kilometres away from the Toronto border. In column (6), we consider instead a spatial trend by interacting the implementation of the LTT with distance to the border. Finally, in column (7), we replicate our preferred specification from column (3) but restrict our sample to all properties sold within two kilometres of the Toronto border instead of three. The coefficient is very close to that in our preferred specification (as well as that from using a five kilometre threshold in columns 4 and 5). In an unreported specification, we also run a robustness check with a "donut" approach. One may be concerned that there might be some sorting of buyers from inside to outside the border in response to the LTT, violating the assumption that neighborhoods right outside of the city border are unaffected by the tax change. Such substitution, if exists, would mostly happen right adjacent to the border. To mitigate this concern, we repeat the main estimation within the 3km (5km) ring but exclude the 1km (2km) that is adjacent to the border from both sides. The results do not change significantly, suggesting that substitution across the border is unlikely to bias our main estimates.

In our preferred specification (column 3), we find that the implementation of the LTT is associated with a 7.9% decrease in sales. This estimate is robust across specifications and consistent with the literature. Using the UK property transaction data, Best and Kleven (2018) find that a temporary 1 percentage-point cut in the tax rate — due to the 2008–9 stamp duty holiday on properties worth between £125,001 and £175,000 — led to a 20% increase in transactions. Using the German single-family home sales, Fritzsche and Vandrei (2019) find that a one-percentage-point increase in the transfer tax yields about 7% fewer transactions. Moreover, Dachis, Duranton and Turner (2012) show that the same LTT studied here caused a 15% decline in the sales volume. The difference between our estimates and theirs is mostly due to the estimation sample. As shown in the appendix, when we restrict the sample to single-family home sales between January 2006 and August 2008 as in Dachis, Duranton and Turner (2012), we find that the LTT caused a 17% decline in sales volume.

Property sales alone can be an imperfect proxy for residential mobility because sales can be undertaken by investors and landlords. For this reason, we now go beyond the literature and use the specifications established above to examine the composition of sales. Take row (2) in Table 1 as an example. Our main specification, column (3), indicates that transactions made by investors increased by 9% in response to the LTT. This estimate is robust across different specifications. It implies that the decrease in sales caused by the LTT is entirely driven by the decline in transactions made by homeowners. Consistent with this, we find that the LTT reduces homeowner transactions by 10% (row 3). Thus, while the LTT substantially reduces residential mobility, it does not discourage investors turnover. Given the dominance of home transactions in the housing market, the total sales volume dropped in response to the LTT, as seen in row (1).

By construction, investors are those who buy to lease. An increase in investor-purchased transactions should lead to an increase in the number of completed leases in the rental sector. Consistent with this, row (4) indicates that the implementation of the LTT is associated with a 16.8% increase in the total number of lease transactions.

Table 1. *Effects of the LTT on market-level outcomes*

	Dep.Variables	[1]	[2]	[3]	[4]	[5]	[6]	[7]
	Transaction market							
(1)	ln(# Total Sales)	-0.099***	-0.075***	-0.079***	-0.100***	-0.100***	-0.070**	-0.096***
		(0.016)	(0.017)	(0.017)	(0.013)	(0.013)	(0.028)	(0.022)
	Observations	7529	7529	7529	12967	12967	12967	5002
(2)	ln(# Investment sales)	0.054	0.092**	0.090**	0.105***	0.105***	-0.015	-0.009
		(0.036)	(0.040)	(0.040)	(0.029)	(0.029)	(0.063)	(0.049)
	Observations	892	892	892	1809	1809	1809	572
(3)	ln(# Home Sales)	-0.065**	-0.022	-0.101**	-0.073***	-0.073***	-0.043	-0.054*
		(0.022)	(0.022)	(0.055)	(0.018)	(0.018)	(0.031)	(0.030)
	Observations	7493	7493	7493	12905	12905	12905	7493
	Leasing market							
(4)	ln(# Leases)	0.141***	0.166***	0.168***	0.155***	0.155***	0.161***	0.173***
		(0.025)	(0.027)	(0.027)	(0.019)	(0.019)	(0.042)	(0.034)
	Observations	2517	2517	2517	4948	4948	4948	1598
(6)	ln(% Owner sales)	-0.004	-0.007**	-0.007**	-0.007**	-0.007**	-0.005	-0.005
		(0.003)	(0.003)	(0.002)	(0.002)	(0.002)	(0.005)	(0.004)
	Observations	7493	7493	7493	12905	12905	12905	4974
(7)	Lease/sales	0.037***	0.035***	0.035***	0.050***	0.050***	0.037**	0.044***
		(0.008)	(0.008)	(0.008)	(0.007)	(0.007)	(0.014)	(0.011)
	Observations	7449	7449	7449	12794	12794	12794	4949
	Price (Invest v.s. Own)							
(8)	Ave. Sales Price	-0.014**	-0.018**	-0.016**	-0.016**	-0.016**	-0.021**	-0.014**
		(0.006)	(0.006)	(0.008)	(0.005)	(0.005)	(0.008)	(0.006)
	Observations	7529	7529	7529	12905	12905	12905	4974
(9)	Ave. Price by Owners	-0.016**	-0.020**	-0.016**	-0.016**	-0.016**	-0.023**	-0.027**
		(0.006)	(0.006)	(0.006)	(0.005)	(0.005)	(0.008)	(0.008)
	Observations	7493	7493	7493	12905	12905	12905	4974
(10)	Ave. Price by Investors	0.045	0.049	0.061	-0.012	-0.012	0.046	0.012
		(0.032)	(0.034)	(0.072)	(0.026)	(0.026)	(0.041)	(0.045)
	Observations	892	892	892	1809	1809	1809	572
(11)	Price by Investors/Price by Owners	0.0732*	0.079*	0.054**	-0.004	-0.004	0.073*	-0.003
		(0.717)	(0.826)	(1.782)	(0.565)	(0.565)	(0.080)	(0.031)
	Observations	856	856	856	1747	1747	1747	544
(12)	Same property price/rent	-1.354*	-1.946**	-1.129	-1.503**	-1.503**	-1.356*	-2.720**
		(0.453)	(0.502)	(0.505)	(0.349)	(0.349)	(0.719)	(1.122)
	Observations	634	634	634	1350	1350	1350	393
	Distance Threshold	3KM	3KM	3KM	5KM	5KM	5KM	2KM
	Indicators TO $\pm$ 3m.	NO	YES	YES	YES	YES	YES	YES
	Time Trends TO	NO	NO	YES	YES	YES	YES	YES
	Distance LTT trends	NO	NO	NO	NO	STEP	YES	NO
	Year* Toronto Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
	Month* Toronto Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
	Year* Month*Property Type Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
	Community*Property Type Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Notes: A unit of observation is a market segment defined by community X property type X year X month. Each cell represents a separate regression of a housing outcome variable (specified on the left column) on the LTT. All regressions are estimated with community fixed effects, year fixed effects, month fixed effects, property type fixed effects and their interactions. House characteristics is a vector of house characteristics discussed in the text. Indicators TO $\pm$ 3m are six dummy variables for transactions inside Toronto during the last 3 months of 2007 and the first 3 of 2008. Time trends TO indicates the presence of separate time trends for transactions in and outside of Toronto. Distance threshold is the maximum distance to the Toronto border for a transaction to be included. Distance LTT trend denotes the inclusion of an interaction term between LTT and distance to the Toronto border. Standard errors clustered by community in parentheses. *, **, ***: corresponding coefficient significant at 10, 5, 1%.

Recall the discussion on the stagnation (2006–2011) and decline (2011–2016) of homeownership rate in the city of Toronto. A natural question to ask is whether this recent pattern of homeownership could be attributed to the LTT. While we do not have housing stock data to construct the monthly homeownership rate, we consider two proxies as a flow version of the homeownership rate. The first is the fraction of homeowner purchases out of the total sales, which dropped by 0.7% in response to the LTT, as indicated by estimates in Row (6). Not surprisingly, while the imposition of the LTT imposes a higher entry cost to owning a home for both investors and homeowners, such cost is higher for homeowners as the tax reduces their expected mobility. The second is the inverse of the lease to sales ratio. Row (7) indicates that the LTT is associated with a 3.5% increase in the lease to sales ratio. Intuitively, the imposition of the LTT makes owning a home less attractive relative to renting, increasing lease to sales ratio. The increase in demand for rental properties encourages the entry of landlords. This is consistent with an inflow of investors induced by the LTT as evidenced in row (2), contributing to a shift of properties from the owner-occupied sector to the rental sector, resulting in a fall in the homeownership rate.

3.2 Time to move and time to sell

The decline in sales volume can result from either a longer length of stay by existing homeowners, or a longer time to sell for current sellers, or both. In this subsection, we examine both possibilities.

We start from the average length of stay that a seller stays in her home before selling it. Table 2 presents transaction level regressions where we control for time-varying housing characteristics in addition to rich interaction of fixed effects.

Row (1) in Table 2 provide estimates from a Weibull hazard model where a unit of observation is each month for each homeowner and an event is whether the homeowner moves or not. Given our interest in the LTT effect, we estimate the moving hazard as a function of the LTT, controlling for the house price when originally bought, house characteristics, community fixed effects, year fixed effects, month fixed effects, property type fixed effects and some of their interactions. As shown in column (1), being subject to the LTT reduces the transaction hazard by 11.6%. In column (2), when we control for 3 months before and after the LTT, the LTT reduces the transaction hazard by 13.7%. The estimate is quite consistent across different specifications. Overall, controlling for the initial home value, the number of months a household has stayed in the current home and time-varying home characteristics, the implementation of the LTT is associated with a 13% increase in the length of stay for existing homeowners conditional on moving. This substantive lock-in effect points to a widening mismatch between homeowners and their current homes. Our findings are consistent with the evidence found in other countries. For example, using the Netherlands housing market data, Van Ommeren and Van Leuvensteijn (2005) find that a 1 percentage-point increase in the value of transaction costs — as a percentage of the value of the residence — decreases residential mobility rates by 8 percent. Using the UK housing market data, Hilber and Lyytikäinen (2017) find that a 2 percentage-point increase in the SDLT reduces the annual rate of mobility by 2.6 percentage points. While their results are based on households' reported mobility in cross-sectional surveys, our results are based on the actual length of stay of homeowners.

Note that the dampened residential mobility among Toronto city residents relative to the suburban residents after the LTT reduces the supply of new listings in the owner-occupied market but increases the supply of listings in the rental sector. The former is supported by a reduction in the home sales as evidenced in row (3) of Table 1. As shown in row (2) of Table Table 1, the implementation of the LTT increases the number of properties purchased by investors and these properties are subsequently put on the rental market for lease. Thus, the LTT is highly distortive in that they not only reduce residential mobility but also shift housing stock from the owner-occupied market to the rental market. This extensive margin across the two markets is a novel contribution of this study.

Turning to the seller time on the market, row (2) of Table 2 presents transaction level regressions where we focus on single family houses and control for time-varying housing characteristics in addition to rich interaction of fixed effects. Row (1) indicates that the implementation of the LTT is associated with a 9.17% increase in the time on the market, indicating that it takes a longer time for sellers to find a potential buyer. That is, conditional on a home being listed, the transaction speed is reduced given the increased transaction cost for potential buyers. Intuitively, a buyer becomes pickier in selecting better matches to reduce the need to move and pay the transaction cost again in the future.

To summarize, the LTT reduces the flows in the ownership market from both sides of the market. On the one hand, it increases the level of mismatch that existing homeowners will tolerate, resulting in fewer homes being put on the market for sale. On the other hand, it imposes an additional transaction cost on potential home buyers, making them pickier in home shopping. As a result, it takes a longer time for a transaction to complete.

3.3 Prices and rents

In light of the evidence in Figures 2, in this section, we investigate the price implications of the LTT using both the market level data and transaction level data.

First, based on the market segment level data underlying Table 1, row (9) indicates that a roughly one-percentage-point increase in the land transfer tax leads to a 1.5% decline in the average sales price. The estimate is robust when we condition out possible anticipation effects, allow for spatially differentiated trends, and relax the 3-kilometres bands to either side of the border. In Table 2, we further estimate the LTT effects on the sales price using the individual transaction level data. Our strategy for estimating the transaction-level price effect of the LTT is similar to the one we employ in the market-level regressions. The advantage of the transaction level data is that we can further control for rich time-varying house characteristics. The estimated LTT effect on property price is 1.40%. When we further restrict the sample to the single family homes, the LTT effect is increased

Table 2. *Effects of the LTT on transaction-level outcomes*

Variable	[1]	[2]	[3]	[4]	[5]	[6]	[7]
Moving Hazard							
LTT	-0.116**	-0.137***	-0.145***	-0.156**	-0.180***	-0.210**	-0.129**
	(0.049)	(0.051)	(0.053)	(0.039)	(0.041)	(0.043)	(0.060)
Observations	4,939,660	4,939,660	4,939,660	8,335,363	8,335,363	8,335,363	3,436,744
ln (Time on the Market for Single Family Homes)							
LLT	0.125**	0.092*	0.307**	0.089**	0.099**	0.099*	0.099*
	(0.0472)	(0.0527)	(0.108)	(0.0410)	(0.049)	(0.062)	(0.063)
Observations	12032	12032	12032	20448	20448	20448	8225
ln (Sales Price for Single Family Homes)							
LLT(% Change)	-1.97***	-1.49**	-1.78***	-1.22**	-1.22**	-1.19*	-1.57**
	(0.509)	(0.568)	(1.16)	(0.459)	(0.552)	(0.692)	(0.670)
Observations	12032	12032	12032	20448	20448	20448	8225
ln (Sales Price for All Properties)							
LLT(% Change)	-1.24***	-1.06**	-1.40***	-1.32***	-1.18**	-0.592	-1.07**
	(0.371)	(0.418)	(0.838)	(0.336)	(0.409)	(0.517)	(0.523)
Observations	17977	17977	17977	31087	31087	31087	11437
House Characteristics	YES	YES	YES	YES	YES	YES	YES
Year FE* Month FE* Property Type FE	YES	YES	YES	YES	YES	YES	YES
Community FE* Property Type FE	YES	YES	YES	YES	YES	YES	YES
Indicators TO ± 3m.	NO	YES	YES	YES	YES	YES	YES
Time Trends TO	NO	NO	YES	YES	YES	YES	YES
Distance Threshold	3KM	3KM	3KM	5KM	5KM	5KM	2KM
Distance LTT trends	NO	NO	NO	NO	STEP	YES	NO

Note: A unit of observation is a transaction. Each cell represents a separate regression. All regressions are transaction level regressions estimated with time-varying house characteristics, community fixed effects, year fixed effects, month fixed effects, property type fixed effects and their interactions. The moving duration model additionally controls for initial home value. House characteristics is a vector of house characteristics discussed in the text. Indicators TO ± 3m are six dummy variables for transactions inside Toronto during the last 3 months of 2007 and the first 3 of 2008. Time trends TO indicates the presence of separate time trends for transactions in and outside of Toronto. Distance threshold is the maximum distance to the Toronto border for a transaction to be included. Distance LTT trend denotes the inclusion of an interaction term between LTT and distance to the Toronto border. Standard errors clustered by community in parentheses. *, **, ***: corresponding coefficient significant at 10, 5, 1%.

to 1.49%, consistent with the estimate from the market-level regressions.

While the LTT reduces the sales price in general, such effect is stronger when the property is purchased by a homeowner. As shown in rows (9) and (10) of Table 1, while the LTT has a significant effect on the sales price paid by homeowners, its effect on the average price paid by investors is statistically insignificant across all specifications. Unlike homeowners, investors do not live in the property and they do not need to sell and pay LTT to buy again if tenants desire to move. As a result, while the LTT imposes an additional constraint on homeowners' expected future mobility when their idiosyncratic matching value with the current home deteriorates, such constraint does not affect investors. This explains why the LTT reduces the price paid by homeowners but not investors.

To further investigate this, row (11) examines the LTT effect on the average price investors were paying relative to homeowners across the GTA. Column (1) indicates that the imposition of the LTT is associated with a 0.07-percentage-point increase in the ratio of the average price paid by investors to the average price paid by homeowners. That is, relative to buyers in suburban areas, average homebuyers in the city pay less than average investors after the LTT. The evidence is again consistent with the notion that the LTT imposes a higher cost for homeowners relative to investors.

In row (13), we turn our attention to the average price-rent ratio, where the price-rent ratio is derived from the same property at approximately the same time. Row (10) indicates that the imposition of the LTT is associated with a 1.35-percentage-point decline in the price-rent ratio. Since the LTT only applies to the owner-occupied housing market, it reduces the sales price more than the rents, resulting in a lower price-rent ratio on average.

To summarize, our findings suggest that the LTT may lead to the misallocation of the housing stock along both the intensive and extensive margins. On the intensive margin, some transactions that would have occurred in the current period are postponed or never happen; some agents who would have been better matched with a new home are now stuck with their old home. On the extensive margin, some agents who would have owned a home now enter the rental market; some properties that would have been bought by homeowners are now bought by investors. To shed light on the underlying mechanisms and welfare implications, we now present a housing search model that takes into account of both the rental and owner-occupied markets simultaneously.

4 A dual ownership and rental markets model of housing

There is a city with two housing markets: an ownership market and a rental market. There is a unit measure of ex-ante identical properties and a measure ϕ of households. There is free entry of buy-to-let investors, who simply represent funds invested in housing: investors could be households living within the city or from outside.

Properties are either up for sale or up for rent or not available in either market. When not for sale or rent, properties are either renter-occupied or owner-occupied. Households are looking for a property to move into or they currently living in a renter-occupied or owner-occupied property without looking to move. If they are looking for a property to move into, they decide whether to search in the rental market or the ownership market. The owner of a property that is neither owner-occupied nor renter-occupied decides whether to let or sell it. Properties can be owned by investors who do not live in them, as well as by those who bought them for owner occupation.

Meetings between participants in both markets are subject to search frictions and are determined by constant-returns-to-scale meeting functions $M^i(b_i, u_i)$, where the index $i \in \{o, l\}$ denotes either the ownership (o) or rental (l) markets. Meetings allow potential owners to view a property and make offers. The variables b_o and b_l are respectively the measures of those looking to buy, including investors, and those looking to rent, and u_o and u_l those looking to sell/let. Market tightness is denoted by $\theta_i \equiv b_i/u_i$, and given the constant-returns-to-scale meeting functions, the meeting rates on the two sides of each market are:

$$\frac{M^i(b_i,u_i)}{b_i} = q_i(\theta_i), \quad \frac{M^i(b_i,u_i)}{u_i} = \theta_i q_i(\theta_i) \quad \text{for} \quad i \in \{o,l\}, \tag{1}$$

where $q_i(\theta_i)$ is the rate at which a buyer/renter views properties in market i, and $\theta_i q_i(\theta_i)$ is the rate at which a property is viewed. The meeting function $M^i(b_i,u_i)$ is increasing in both b_i and u_i, and hence $q_i(\theta_i)$ decreases with θ_i while $\theta_i q_i(\theta_i)$ increases with θ_i. Intuitively, if there are more buyers relative to sellers, the viewing rate is lower for buyers but higher for properties.

A property generates a match-specific flow value for the household that lives there, but are all the same to investors. At the time of a meeting (viewing), a household draws a match-specific quality ε from a distribution function $G_i(\varepsilon)$ for market i. The distribution can differ across the two markets.

Match quality ε is subject to idiosyncratic shocks representing life events that make a property less well matched to a household than it was initially. Such shocks arrive at rate a_i, which can differ across the two types of housing tenure $i \in \{o,l\}$. For owner-occupiers, match quality is reduced from its previous value ε to $\delta_o\varepsilon$ following a shock, where $\delta_o < 1$. For renters, match quality ε is reduced to 0 following a shock — effectively $\delta_l = 0$. The decision to move is endogenous, depending on how low match quality has become, though for renters, moving depends only on the arrival of the idiosyncratic shock.[14]

There is exogenous exit of households from the city at rate ρ, and an equal inflow of new households. Homeowners exiting the city sell their properties. When tenants exit the city, their landlords look for new tenants. New entrants to the city draw an idiosyncratic cost χ of becoming a homeowner from a distribution $G_m(\chi)$ and then decide whether to rent or to buy. A household becomes a buyer if the cost is below a threshold $\chi \leq z$. The idiosyncratic homeownership cost χ is a persistent variable, but is redrawn by renters with probability ς when they receive a moving shock. The cost χ can be thought of as household-specific factors that affect the cost or availability of credit.

4.1 The ownership market

There are two kinds of buyers in the ownership market: investors and home-buyers. An investor buys a property to let in the rental market, while a home-buyer looks for a property to own and live in. Both homeowners and investors can decide to sell a property following a shock, and if they do so, they have the same expected value from selling because properties are ex ante identical. The value of owning a property is the same for all investors as they aim to let the property and all face

[14] It is possible to extend the model to have $\delta_l > 0$. However, it turns out that the endogeneity of moving is quantitatively unimportant for renters here, so the model is simplified by assuming $\delta_l = 0$.

the same expected rents. However, the value of a property to an owner-occupier depends on match-specific quality ε, the initial value of which is revealed by a viewing. After a meeting, buyers and sellers negotiate a price and a transaction occurs if mutually agreeable. There are proportional taxes on the transaction price at rate τ_h for owner-occupiers and τ_k for investors. After a transaction, an owner-occupier moves into the property, while an investor puts it up to let in the rental market.

The Bellman equation for the value K of being an investor who anticipates paying price P_k is

$$rK = -F_k + q_o(\theta_o)\left(U_l - (1+\tau_k)P_k - C_k - K\right) + \dot{K}, \tag{2}$$

where r is the discount rate for future payoffs, $\dot{K}$ is the derivative of the value K with respect to time t, F_k is the flow search cost of an investor, U_l is the value of having a property to let in the rental market, $\tau_k P_k$ is the tax paid, and C_k is other transaction costs faced by investors.

The Bellman equation for the value B_o of being a home-buyer is

$$rB_o = -F_h + q_o(\theta_o)\int \max\left\{H(\varepsilon) - C_h - (1+\tau_h)p(\varepsilon) - B_o, 0\right\} \mathrm{d}G_o(\varepsilon) - \rho B_o + \dot{B}_o, \tag{3}$$

where $p(\varepsilon)$ is the price paid when match quality is ε, $H(\varepsilon)$ is the value of being an owner-occupier with match quality ε, F_h is the flow search cost, $\tau_h p(\varepsilon)$ is the tax paid, and C_h is other transaction costs such as moving costs. Match quality ε is drawn from the distribution $G_o(\varepsilon)$ on viewing, and then the buyer decides whether to go ahead with the purchase.

The value U_o of having a property to sell satisfies the Bellman equation

$$rU_o = -M + \theta_o q_o(\theta_o)\Bigg((1-\xi)\int \max\left\{p(\varepsilon) - C_u - U_o, 0\right\} \mathrm{d}G_o(\varepsilon) + \xi \max\left\{P_k - C_u - U_o, 0\right\}\Bigg) + \dot{U}_o, \tag{4}$$

where M is the cost of maintaining a property paid by all owners, C_u is the transaction cost paid by the seller, and ξ is the fraction of buyers who are investors. The owner chooses whether to sell, and anticipates prices P_k if selling to an investor and $p(\varepsilon)$ to a home-buyer with match quality ε.

The Bellman equation for the value function $H(\varepsilon)$ of an owner-occupier with match quality ε is

$$rH(\varepsilon) = \varepsilon - M + a_o\left(\max\left\{H(\delta_o\varepsilon), B_o + U_o\right\} - H(\varepsilon)\right) + \rho\left(U_o - H(\varepsilon)\right) + \dot{H}(\varepsilon), \tag{5}$$

where ε is the flow utility derived from occupying a property with match quality ε. The arrival rate of idiosyncratic shocks to match quality is a_o. After a shock, an owner-occupier decides whether to remain in the property and receive value $H(\delta_o\varepsilon)$, or to move out and become a seller (value U_o) and buyer (value B_o) in the ownership market.[15] At rate ρ, owner-occupiers have to leave the city (value

[15]Homeowners cannot become landlords after deciding to move, implicitly because there is a sufficiently large credit cost of having two mortgages to keep their existing property as well as buying a new one.

zero), in which case their property is put up for sale, but they do not look for a property to buy.

4.2 The rental market

Participants on both sides of the rental market — potential tenants and landlords — are ex ante identical. When a meeting (viewing) occurs, the potential tenant draws match quality ε with the property and the two sides negotiate a rent. If a rent is agreed, the owner becomes a landlord and the tenant moves in. After moving in, the tenant faces idiosyncratic shocks with arrival rate a_l that reduce match quality to zero. When a shock occurs, there is a probability ς that a new cost χ of becoming a homeowner is drawn. The tenant then moves out and decides again whether to buy in the ownership market or to look for another property to rent. After a tenant moves out, the landlord decides whether to look for another tenant or sell the property.

The Bellman equation for the value U_l of having a property to let (potential landlord) is

$$rU_l = -M + \theta_l q_l(\theta_l) \int \max\{L(\varepsilon) + T(\varepsilon) - C_l - U_l, 0\} \mathrm{d}G_l(\varepsilon) + \dot{U}_l\,, \tag{6}$$

where an owner who becomes a landlord to a tenant with match quality ε has value $L(\varepsilon)$ after paying transaction cost C_l and receiving a fee $T(\varepsilon)$ from the tenant. The Bellman equation for $L(\varepsilon)$ is

$$rL(\varepsilon) = R(\varepsilon) - M - M_l + (a_l + \rho)\,(\max\{U_l, U_o\} - L(\varepsilon)) + \dot{L}(\varepsilon)\,, \tag{7}$$

where $R(\varepsilon)$ is the rent and M_l is an extra maintenance cost incurred when properties are let. Idiosyncratic shocks that lead the tenant to move out occur at combined rate $a_l + \rho$.

The value B_l of a potential tenant searching to rent a property satisfies the Bellman equation

$$rB_l = -F_w + q_l(\theta_l) \int \max\{W(\varepsilon) - T(\varepsilon) - C_w - B_l, 0\}\, \mathrm{d}G_l(\varepsilon) - \rho B_l + \dot{B}_l\,, \tag{8}$$

where F_w is the flow search cost of the potential tenant, $W(\varepsilon)$ is the value function for a tenant with match quality ε, and $T(\varepsilon)$ and C_w are the fee and own transaction costs paid by the tenant. The Bellman equation for the value function $W(\varepsilon)$ is

$$rW(\varepsilon) = \varepsilon - R(\varepsilon) + a_l(1-\varsigma)(B_l - W(\varepsilon)) + a_l\varsigma\,(G_m(z)(B_o - \bar{z}) + (1 - G_m(z))B_l - W(\varepsilon)) \\ - \rho W(\varepsilon) + \dot{W}(\varepsilon)\,. \tag{9}$$

Shocks with arrival rate a_l make the tenant move out. With probability $1-\varsigma$, the credit cost χ is not redrawn and the household re-enters the rental market (value B_l). If a new value of χ is drawn then there is a cut-off z below which it is optimal for the household to buy in the ownership market (value B_o). The term $\bar{z} \equiv E[\chi|\chi \le z]$ denotes the average cost χ conditional on entering the ownership market. New entrants to the city choose to enter the ownership or rental markets in exactly the same way as tenants who redraw the cost χ.

4.3 Entry and transitions between the two markets

The free entry condition for investors holds at all points in time:

$$K = 0\,. \tag{10}$$

There is a positive cost threshold z satisfying the indifference condition between the two markets:

$$z = B_o - B_l\,. \tag{11}$$

4.4 Functional forms and parameter restrictions

The meeting functions $M^i(b_i, u_i)$ have Cobb-Douglas functional forms

$$M^i(b_i, u_i) = A_i b_i^{1-\eta_i} u_i^{\eta_i}\,, \quad \text{hence} \quad q_i(\theta_i) = A_i \theta_i^{-\eta_i}\,, \tag{12}$$

where A_i is the efficiency with which meetings occur in market i, and η_i is the elasticity of buyers' meeting rate with respect to market tightness θ_i. These parameters can differ across markets. New match quality ε is drawn from Pareto distributions

$$G_i(\varepsilon) = 1 - \left(\frac{\varepsilon}{\zeta_i}\right)^{-\lambda_i} \quad \text{for} \quad i \in \{\text{o}, \text{l}\}\,, \quad \text{and where} \quad \lambda_i > 1\,, \tag{13}$$

with ζ_i being the minimum possible draw in market i, and λ_i specifying the shape of the distribution, specifically how compressed realizations of ε are towards the minimum. The expected match quality from a viewing is $E_i[\varepsilon] = \zeta_i \lambda_i / (\lambda_i - 1)$. Draws of the homeownership cost χ are from a log Normal distribution with mean and standard deviation parameters μ and σ:

$$G_m(\chi) = \Phi\left(\frac{\log \chi - \mu}{\sigma}\right)\,, \tag{14}$$

where $\Phi(\cdot)$ is the standard Normal CDF. The average level of χ conditional on $\chi \leq z$ is

$$\bar{z} = e^{\mu + \frac{\sigma^2}{2}} \frac{\Phi\left(\frac{\log z - \mu - \sigma^2}{\sigma}\right)}{\Phi\left(\frac{\log z - \mu}{\sigma}\right)}\,. \tag{15}$$

It has already been stated that $\delta_l = 0$, so moving is exogenous in the rental market. Attention is also restricted to δ_o sufficiently low that some, but not all, ownership-market matches require only one idiosyncratic shock to trigger moving.

4.5 Equilibrium in both housing markets

This section studies the transaction and moving decisions in the ownership and rental markets, and the allocation of properties and households across the two markets implied by the free entry (10) and indifference condition (11). These decisions determine transaction volumes in each of the markets, property prices, rents, and the homeownership rate. Finally, the equilibrium conditions are completed with the laws of motion for the stocks of properties and households in the two markets.

4.5.1 Ownership market decisions

Consider the transaction decision when a home-buyer meets a seller. Property prices are determined by Nash bargaining with seller bargaining power ω_o. The surpluses in the negotiation of $p(\varepsilon)$ between a home-buyer and a seller are

$$\Sigma_o^h(\varepsilon) = H(\varepsilon) - (1+\tau_h)p(\varepsilon) - C_h - B_o\,, \quad \text{and} \quad \Sigma_o^u(\varepsilon) = p(\varepsilon) - C_u - U_o\,, \tag{16}$$

and the joint surplus of the two parties is

$$\Sigma_o(\varepsilon) \equiv \Sigma_o^h(\varepsilon) + \Sigma_o^u(\varepsilon) = H(\varepsilon) - C_h - C_u - B_o - U_o - \tau_h p(\varepsilon)\,. \tag{17}$$

For any given level match quality ε, the Nash bargaining problem is to choose $p(\varepsilon)$ to maximize $(\Sigma_o^u(\varepsilon))^{\omega_o}\left(\Sigma_o^h(\varepsilon)\right)^{1-\omega_o}$, where the individual surpluses must be non-negative. Using (16), the first-order condition implies the surplus sharing equation

$$\frac{\Sigma_o^u(\varepsilon)}{\Sigma_o^h(\varepsilon)} = \frac{\omega_o}{(1-\omega_o)(1+\tau_h)}\,. \tag{18}$$

In the absence of the transaction tax τ_h, the surplus would have been divided according to bargaining powers in line with the usual Nash rule. A positive transaction tax skews the sharing of the surplus in favour of the buyer. Intuitively, the joint surplus is increased by agreeing a lower price because of the proportional tax, and a lower price increases the buyer's surplus. The resulting split is

$$\Sigma_o^h(\varepsilon) = (1-\omega_o^*)\Sigma_o(\varepsilon) \quad \text{and} \quad \Sigma_o^u(\varepsilon) = \omega_o^*\Sigma_o(\varepsilon)\,, \quad \text{where} \quad \omega_o^* \equiv \frac{\omega_o}{1+\tau_h(1-\omega_o)}\,. \tag{19}$$

The seller's share of the surplus ω_o^* is below her bargaining power ω_o.

When the joint surplus is positive a transaction occurs with price $p(\varepsilon)$ that delivers the equilibrium surplus shares (19) in (16):

$$p(\varepsilon) = C_u + U_o + \omega_o^*\Sigma_o(\varepsilon)\,. \tag{20}$$

With this price, the equilibrium joint surplus in (17) becomes

$$\Sigma_o(\varepsilon) = \frac{H(\varepsilon) - C_h - B_o - (1+\tau_h)(C_u + U_o)}{1 + \tau_h \omega_o^*} \,. \tag{21}$$

Given that match quality ε is observable and surplus is transferable, transactions go ahead when the match quality is above a threshold y_o where the joint surplus is zero:

$$\Sigma_o(y_o) = 0 \,. \tag{22}$$

Conditional on a viewing, the expected surplus and transaction probability are

$$\Sigma_o \equiv \int_{y_o} \Sigma_o(\varepsilon) \mathrm{d}G_o(\varepsilon) \,, \quad \text{and} \quad \pi_o(y_o) \equiv \int_{y_o} \mathrm{d}G_o(\varepsilon) \,. \tag{23}$$

The average transaction price P is obtained from (20) using the terms in (19):

$$P \equiv \frac{1}{\pi_o} \int_{y_o} p(\varepsilon) \mathrm{d}G_o(\varepsilon) = \frac{\omega_o^* \Sigma_o}{\pi_o} + C_u + U_o \,. \tag{24}$$

Given possibility of moving in the future, the transaction threshold and the joint surplus depend not only on transactions decisions but also on moving decisions.

The value function (5) can be used to define the moving threshold x_o in the ownership market:

$$H(x_o) = B_o + U_o \,. \tag{25}$$

The condition that some matches require only one idiosyncratic shock to trigger moving is equivalent to $\delta_o y_o < x_o$.

4.5.2 Rental market decisions

While a tenant lives in a property with match quality ε, the rent $R(\varepsilon)$ is determined by Nash bargaining with landlord bargaining power ω_l. Tenant and landlord surpluses are

$$S^w(\varepsilon) = W(\varepsilon) - B_l \,, \quad \text{and} \quad S^l(\varepsilon) = L(\varepsilon) - U_l \,, \tag{26}$$

which assumes $U_l \geq U_o$, as will be confirmed. The joint surplus in the rental market is:

$$S(\varepsilon) \equiv S^w(\varepsilon) + S^l(\varepsilon) = W(\varepsilon) + L(\varepsilon) - B_l - U_l \,. \tag{27}$$

Note that the transaction costs C_l and C_w have already been paid and are sunk at the time of the rent negotiation, and that the bargaining problem for new rental contracts is the same as for continuing rental contracts because the tenant's cost χ of becoming a homeowner does not change except after a moving shock. Nash bargaining implies the simple Nash sharing rule whereby the joint surplus is split between the tenant and the landlord according to their bargaining powers:

$$S^w(\varepsilon) = (1-\omega_l)S(\varepsilon)\,, \quad \text{and} \quad S^l(\varepsilon) = \omega_l S(\varepsilon)\,. \tag{28}$$

Now consider a meeting between a potential tenant and landlord where the match quality is ε:

$$\Sigma_l^w(\varepsilon) = W(\varepsilon) - T(\varepsilon) - C_w - B_l\,, \quad \text{and} \quad \Sigma_l^l(\varepsilon) = L(\varepsilon) + T(\varepsilon) - C_l - U_l\,. \tag{29}$$

The total surplus is

$$\Sigma_l(\varepsilon) \equiv \Sigma_l^w(\varepsilon) + \Sigma_l^l(\varepsilon) = W(\varepsilon) + L(\varepsilon) - C_w - C_l - B_l - U_l = S(\varepsilon) - (C_w + C_l)\,, \tag{30}$$

where $S(\varepsilon)$ is the ongoing total surplus from (27) that is divided in rent negotiations after the costs C_w and C_l are incurred. Nash bargaining over the initial fee $T(\varepsilon)$ implies joint surplus is split between the tenant and the landlord according to their bargaining powers:

$$\Sigma_l^w(\varepsilon) = (1-\omega_l)\Sigma_l(\varepsilon)\,, \quad \text{and} \quad \Sigma_l^l(\varepsilon) = \omega_l \Sigma_l(\varepsilon)\,. \tag{31}$$

Since $\Sigma_l^w(\varepsilon) = S^w(\varepsilon) - T(\varepsilon) - C_w$ and $\Sigma_l^l(\varepsilon) = S^l(\varepsilon) + T(\varepsilon) - C_l$ from (26) and (29), equations (28) and (31) imply the fee paid by the tenant to the landlord is independent of match quality ε:

$$T(\varepsilon) = T = (1-\omega_l)C_l - \omega_l C_w. \tag{32}$$

The transaction threshold y_l is where the joint surplus is zero:

$$\Sigma_l(y_l) = 0\,, \tag{33}$$

and a transaction occurs if the match quality exceeds the transaction threshold. Conditional on a viewing, the expected surplus and transaction probability in the rental market are

$$\Sigma_l \equiv \int_{y_l} \Sigma_l(\varepsilon)\mathrm{d}G_l(\varepsilon)\,, \quad \text{and} \quad \pi_l(y_l) \equiv \int_{y_l} \mathrm{d}G_l(\varepsilon)\,. \tag{34}$$

The average rent R for new matches is

$$R = \frac{\int_{y_l} R(\varepsilon)\mathrm{d}G_l(\varepsilon)}{\pi_l}\,. \tag{35}$$

4.5.3 Investor decisions

If an investor meets a seller in the ownership market then their surpluses are respectively

$$\Sigma_k^k = U_l - (1+\tau_k)P_k - C_k - K\,, \quad \text{and} \quad \Sigma_k^u = P_k - C_u - U_o\,, \tag{36}$$

and their joint surplus is

$$\Sigma_k \equiv \Sigma_k^k + \Sigma_k^u = U_l - C_k - C_u - U_o - K - \tau_k P_k\,. \tag{37}$$

Similar to the Nash bargaining between a seller and a home-buyer, a transaction takes place when the total surplus is positive, and the joint surplus is shared between investor and seller according to

$$\frac{\Sigma_k^u}{\Sigma_k^k} = \frac{\omega_k}{(1-\omega_k)(1+\tau_k)}\,. \tag{38}$$

The surpluses of the investor and the seller are therefore:

$$\Sigma_k^k = (1-\omega_k^*)\Sigma_k \quad \text{and} \quad \Sigma_k^u = \omega_k^*\Sigma_k\,, \quad \text{where} \quad \omega_k^* \equiv \frac{\omega_k}{1+\tau_k(1-\omega_k)}\,. \tag{39}$$

Note from the joint surplus equation (37), a positive joint surplus implies that the value of having a property to let is always above the value of having a property for sale, that is, $U_l \geq U_o$. Thus, after purchasing a property, an investor strictly prefers to let it out in the rental market.[16]

Using (36) and (39), the price paid by investors is

$$P_k = C_u + U_o + \omega_k^*\Sigma_k\,, \tag{40}$$

and substituting this into (37) shows that the joint surplus is

$$\Sigma_k = \frac{U_l - (1+\tau_k)U_o - (1+\tau_k)C_u - C_k}{1+\tau_k\omega_k^*}\,. \tag{41}$$

Given the free-entry condition (10), the Bellman equation (2) for the investor value implies

$$\Sigma_k = \frac{F_k}{(1-\omega_k^*)q_o(\theta_o)}\,, \tag{42}$$

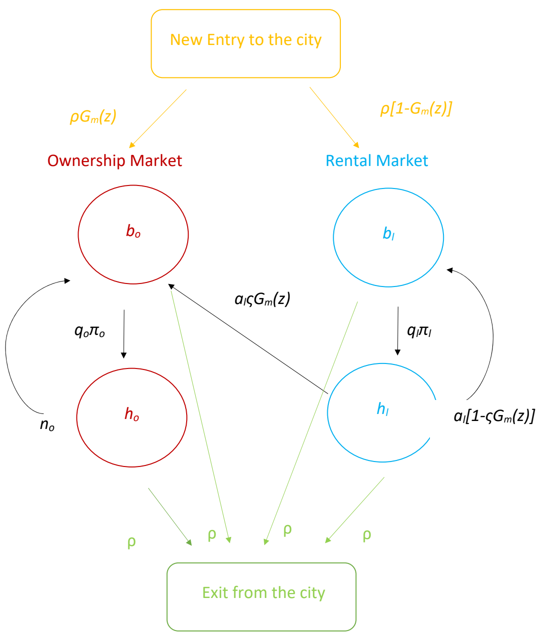
which shows the expected total surplus is an increasing function of market tightness in the ownership market. Intuitively, when the market tightness θ_o goes up (more buyers relative to sellers), the meeting rate $q_o(\theta_o)$ decreases for an investor. Thus the free entry condition requires that an investor has to be compensated by a higher expected surplus $(1-\omega_k^*)\Sigma_k$ to enter the market.

[16]In other words, pure 'flippers' — those who buy and sell immediately — are not present in the model.

4.5.4 Stocks and flows

The flows and stocks in the ownership and rental markets are summarized in Figure 5 for households. A new household in the city chooses to search in the ownership market or the rental market. Households in the ownership market can be looking to buy or living in an owner-occupied property. If an owner-occupier household decides to move, the household searches in the ownership market again. Households in the rental market are either looking to rent or living in a renter-occupied property. When a renter-occupier household moves, the household decides whether to rent again or pay the credit cost and look for a property to buy. All households in the city exit at rate ρ.

Figure 5: *Flows and stocks*



A property can be in any of four states: owner-occupied (with measure h_o), renter-occupied (with measure h_l), for rent (with measure u_l) or for sale (with measure u_o). They must sum up to the measure of total properties:

$$h_o + h_l + u_o + u_l = 1\,. \tag{43}$$

Similarly, a household can be in any one of four states: owner-occupier, renter-occupier, looking for property to rent, or looking for property to buy. Assuming one family can only occupy one property, thus h_o and h_l are also the measures of families who are owner-occupiers and renter-occupiers, respectively. Together they sum up to the total measure of households ϕ:

$$h_o + h_l + b_{oo} + b_l = \phi \,. \tag{44}$$

Buyers b_{oo} looking to become owner-occupiers are a fraction $1-\xi$ of total buyers $b_o = b_{oo} + b_k$, where b_k denotes investors. The homeownership rate h is defined by:

$$h = h_o + u_o \,. \tag{45}$$

Let s_i denote the sales/letting rate and n_i denote the moving rates in the two market. The law of motion for the stock of properties for sale is

$$\dot{u}_o = (n_o + \rho)h_o - s_o u_o \,, \tag{46}$$

where s_o is a weighted average of the sales rates to owner-occupiers s_{oo} and investors s_k:

$$s_o = (1-\xi)s_{oo} + \xi s_k \,. \tag{47}$$

The law of motion for the stock of properties to let is:

$$\dot{u}_l = (n_l + \rho)h_l + \xi s_k u_o - s_l u_l \,. \tag{48}$$

The law of motion for owner-occupied properties is

$$\dot{h}_o = (1-\xi)s_{oo}u_o - (n_o + \rho)h_o \,, \tag{49}$$

and for renter-occupied properties:

$$\dot{h}_l = s_l u_l - (n_l + \rho)h_l \,. \tag{50}$$

The law of motion for those looking to rent is:

$$\dot{b}_l = a_l(1-\varsigma)h_l + (\rho\phi + a_l\varsigma h_l)(1 - G_m(z)) - (q_l(\theta_l)\pi_l(y_l) + \rho)b_l \,, \tag{51}$$

where inflows are from previous renter-occupiers $a_l(1-\varsigma)h_l$ who do not redraw their credit cost and new entrants to the city and existing renters who draw credit costs above the threshold for becoming a homeowner $(\rho\phi + a_l\varsigma h_l)(1-G_m(z))$. The law of motion for home-buyers is

$$\dot{b}_{oo} = n_o h_o + (\rho\phi + a_l\varsigma h_l)G_m(z) - (q_o(\theta_o)\pi_o(y_o) + \rho)b_{oo} \,, \tag{52}$$

where inflows are from those who move within the ownership market $n_o h_o$, and new entrants to the city and previous renters who draw new credit costs below the threshold $(\rho\phi + a_l \varsigma h_l)G_m(z)$.

The moving and transaction rates are endogenously determined by the moving decisions of owner-occupiers and renter-occupiers, and the transactions decisions of individual buyers, sellers, renters and landlords:

$$s_o = \theta_o q_o(\theta_o)\pi_o(y_o)\,, \quad s_k = \theta_o q_o(\theta_o)\,, \quad \text{and} \quad s_l = \theta_l q_l(\theta_l)\pi_l(y_l)\,, \tag{53}$$

where the meeting rate $\theta_i q_i(\theta_i)$ captures the search friction in locating properties. The term $\pi_i(y_i)$ is the proportion of viewings for which match quality is above the transactions threshold, capturing the search friction due to idiosyncratic tastes. The Pareto distributions of new match quality (13) imply the acceptance probabilities are:

$$\pi_i(y_i) = \left(\frac{y_i}{\zeta_i}\right)^{-\lambda_i}, \quad \text{for} \quad i \in \{o, l\}\,. \tag{54}$$

The endogenous moving rate n_o is derived from the distribution of existing match quality among owner-occupiers together with the moving threshold x_o. The evolution over time of the distribution of match quality depends on idiosyncratic shocks and moving decisions. Surviving matches differ along two dimensions: (i) the initial level of match quality and (ii) the number of shocks received since the match formed. By using the Pareto distribution (13) for new match quality, the appendix shows that the endogenous moving rate is

$$n_o = a_o - \frac{a_o \delta_o^{\lambda_o} x_o^{-\lambda_o}}{h_o} \int_{\tau \to -\infty}^{t} e^{-a_o(1-\delta_o^{\lambda_o})(t-\tau)} e^{-\rho(t-\tau)} \theta_o q_o(\theta_o) u_o \mathrm{d}\tau\,. \tag{55}$$

This equation shows that given the moving threshold x_o, the endogenous moving rate n_o displays history dependence due to the persistence in the distribution of match quality among existing occupiers. For the rental market, the moving rate is simply

$$n_l = a_l\,. \tag{56}$$

4.6 Steady state

In an equilibrium with (10), it can be shown the value of having a property for rent is larger than the value of having a property for sale, that is, $U_l \geq U_o$, so landlords have no incentive to exit the market. Hence, there exists a steady-state equilibrium with no entry of new investors, that is, the fraction ξ of investors among buyers is zero. The past entry of investors forms the stock of investors in the steady state, which determines the steady-state homeownership rate.

In a steady state with $\dot{h}_o = 0$ and $\dot{h}_l = 0$, the laws of motion (49) and (50) for owner-occupied and renter-occupied properties imply

$$s_o u_o = (n_o + \rho) h_o \,, \tag{57}$$

and

$$s_l u_l = (n_l + \rho) h_l \,. \tag{58}$$

Expected viewings per sale and per lease are:

$$V_o = \frac{1}{\pi_o} \quad \text{and} \quad V_l = \frac{1}{\pi_l}\,, \tag{59}$$

and the expected times taken to sell, buy, let, and rent are:

$$T_{so} = \frac{1}{s_o} \quad , \quad T_{bo} = \frac{1}{q_o \pi_o} \quad , \quad T_{sl} = \frac{1}{s_l} \quad , \quad T_{bl} = \frac{1}{q_l \pi_l}\,. \tag{60}$$

The expected lengths of stay in a property for owners and renters are:

$$T_{mo} = \frac{1}{n_o + \rho} \quad \text{and} \quad T_{ml} = \frac{1}{n_l + \rho}\,. \tag{61}$$

The steady-state endogenous moving rate for owner-occupiers is derived in the appendix from (46), (49), and (55) using $\dot{u}_o = 0$ and $\dot{n}_o = 0$:

$$n_o = a_o \left(\frac{1 - \frac{\rho \delta_o^{\lambda_o} \left(\frac{y_o}{x_o}\right)^{\lambda_o}}{\rho + a_o\left(1 - \delta_o^{\lambda_o}\right)}}{1 + \frac{a_o \delta_o^{\lambda_o} \left(\frac{y_o}{x_o}\right)^{\lambda_o}}{\rho + a_o\left(1 - \delta_o^{\lambda_o}\right)}} \right) = a_o \left(\frac{\rho + a_o\left(1 - \delta_o^{\lambda_o}\right) - \rho \delta_o^{\lambda_o} \left(\frac{y_o}{x_o}\right)^{\lambda_o}}{\rho + a_o\left(1 - \delta_o^{\lambda_o}\right) + a_o \delta_o^{\lambda_o} \left(\frac{y_o}{x_o}\right)^{\lambda_o}} \right), \tag{62}$$

which is decreasing in ρ. Intuitively, the presence of ρ is making the cohort of existing of homeowners younger, so they are likely to have higher existing match quality, thus a lower endogenous moving rate.

Observe that there is a positive relationship between the moving threshold x_o and the moving rate n_o in the ownership market. The transaction threshold y_o appears in the equation and has a negative effect because it is positively related to average initial match quality. In the special case where λ_o is very large and the distribution $G_o(\varepsilon)$ is degenerate, the effect of thresholds on the moving rate disappears and only the arrival rate a_o of the exogenous shock will matter. Intuitively, if everyone has the same match quality then everyone who gets a shock has to move otherwise there will be no moving. Put it differently, there is no marginal homeowner. Thus a distribution of match quality is an essential ingredient in making the moving rate endogenous.

The law of motion (51) for those looking to rent with $\dot{b}_l = 0$ implies

$$a_l(1-\varsigma)h_l + (\rho\phi + a_l\varsigma h_l)(1-G_m(z)) = (q_l(\theta_l)\pi_l(y_l) + \rho)b_l\,, \tag{63}$$

so using the definition of market tightness $\theta_l = b_l/u_l$, the equilibrium cost threshold z satisfies:

$$1 - G_m(z) = \frac{(q_l(\theta_l)\pi_l(y_l) + \rho)\theta_l u_l - a_l(1-\varsigma)h_l}{a_l\varsigma h_l + \rho\phi}\,. \tag{64}$$

Substituting the steady-state conditions (57) and (58) into the stock of properties (43) and using (A.35), the steady-state stock of property for rent is:

$$u_l = \frac{1 - \left(1 + \frac{s_o}{n_o+\rho}\right)\left(\frac{\phi-1}{\theta_o-1}\right)}{\left(1 + \frac{s_o}{n_o+\rho}\right)\left(\frac{1-\theta_l}{\theta_o-1}\right) + \left(1 + \frac{s_l}{n_l+\rho}\right)}\,. \tag{65}$$

Given u_l, the steady-state level of u_o is derived from (43) and (44):

$$u_o = \frac{\phi - 1 + (1-\theta_l)u_l}{\theta_o - 1}\,. \tag{66}$$

5 Quantitative results

In Toronto, both the homebuyers and buy-to-let investors face the same LTT when purchasing a property, thus τ_k is the same as τ_h before and after the LTT increase. Interestingly, as discussed in the empirical section 3, despite facing the same taxes and same increase, transactions rose for buy-to-let investors and fell for homebuyers. Such responses are tightly linked to the observed fall in price-to-rent ratio. The model illustrates the importance of allowing for the flows between the ownership and rental markets to understand these responses and we now turn to its quantitative implications.

We calibrate the model to match some key features of ownership and rental markets in the city of Toronto before the LTT change. The LTT for the city of Toronto was introduced in February 2008 so any transactions after such date have to pay the new Toronto tax in addition to the original provincial tax.[17] The effective LTT rate, measured by the mean of land transfer taxes relative to home sales prices across transactions, is 1.08% during the pre-policy period and 2.04% during the post-policy period for the city of Toronto, which implies an increase of 0.96%.

Table 3. *Calibration targets*

Targets	Notation	Value
Directly set targets		
Normalization of minimum new match quality in ownership market	ζ_o	1
Equal numbers of households and properties	ϕ	1
Equal landlord and tenant bargaining power	ω_l	0.5
Equal bargaining power for sellers facing home-buyers and investors	ω_k/ω_o	1
Elasticity of matching function equal to bargaining power	ω_o/η_o=ω_l/η_l	1
Data targets		
Effective land transfer tax for buyers	$\tau_k = \tau_h$	0.011
Transaction costs of buyers excluding tax relative to price	$C_h/P = C_k/P$	0
Maintenance cost relative to price	M/P	0.026
Extra maintenance/management costs of landlords relative to rent	M_l/R	0.08
Seller transaction costs relative to price	C_u/P	0.045
Landlord transaction costs relative to rent	C_l/R	0.083
Tenant transaction costs relative to rent	C_w/R	0.083
Flow search costs of investors relative to price	F_k/P	0.040
Flow search costs of home-buyers relative to price	F_h/P	0.040
Flow search costs of tenants relative to home-buyers	F_w/F_h	0.942
Sellers' time on the market	T_{so}	0.127
Buyers' time on the market	T_{bo}	0.162
Landlords' time on rental market	T_{sl}	0.066
Viewings per sale	V_o	20.6
Viewings per lease	V_l	10.3
Average time between moves for owner-occupiers	T_{mo}	5.51
Average time between moves for tenants	T_{ml}	3.04
Average price-rent ratio for same property	P/R	14.8
Homeownership rate	h	0.54
Capitalized credit costs of marginal buyer relative to price	z/P	0.39
Fraction of first-time buyers	ψ	0.4
Difference in average age of owner-occupiers and renters	γ	8.3
Ratio of credit costs of marginal and median buyers" notation	$z/\hat{z}$	2.09
Elasticity of owner-occupier moving rate to tax change	β	10

Notes: See appendix A.4 for data sources and appendix A.5 for the calibration procedure.

5.1 Calibration

The model is calibrated to the Toronto housing market before the LTT change, i.e. during the 2007–2008 period. The tax rate faced by both the home-buyer and buy-to-let investor are set to the effective

[17]See Appendix Table 7 for the rates. Dachis, Duranton and Turner (2012) provide a brief history of the introduction of LTT and argue that the LTT change was largely unanticipated.

LTT prior to the change, $\tau_k = \tau_h = 0.011$. The parameters of the model are calibrated to match a list of targets in Table 3 and the implied parameter values are reported in Table 4. The data source of all targets can be found in appendix A.4, and appendix A.5 provides the detailed calibration procedure. In summary, there are three broad sets of targets.

The first set of targets and parameters are set directly: $\left(\zeta_o, \phi, \omega_l, \frac{\omega_k}{\omega_o}, \frac{\omega_o}{\eta_o}, \frac{\omega_l}{\eta_l}\right)$. The minimum new match quality in the ownership market ζ_o is normalized to 1 and the measure of households is set to be the same as the measure of properties, $\phi = 1$. There is little information available for the bargaining power in the housing market. Our strategy is to assume equal bargaining power between landlord and tenant, $\omega_l = 0.5$. The bargaining power of the sellers is set to be the same whether he is facing a home-buyers or a buy-to-let investors, $\omega_k = \omega_o$. Finally the bargaining power of sellers (landlord) are set as the same as their corresponding elasticity in the matching functions of the two market, $\omega_o = \eta_o, \omega_l = \eta_l$.

The second set of targets are set to match the search behavior and associated costs in the housing markets. The key targets for search behavior are viewings per sale V_o, viewings per lease V_l, time on the market for buyers and sellers T_{so}, T_{bo} in the ownership market and landlord's time on the rental market T_{sl}; and time-to-move for both homeowners and tenants T_{mo}, T_{ml}. The targets for costs in the ownership markets are maintenance cost for homeowner M, transaction costs excluding taxes for buyers and sellers (C_k, C_h, C_u), search flow costs of buyers F_k, F_h, all as a fraction of price. The target for costs in the rental markets include the extra maintenance costs M_l, transaction costs of landlords and renters (C_l, C_w) and search flow costs of tenant F_w, as a fraction of rent.

The last set of targets are related to the extensive margin across ownership and rental markets. They are the homeownership rate h, price-to-rent ratio P/R, the capitalized credit costs of marginal buyers relative to price z/P, the fraction of first-time buyers ψ and the difference in the average age of owner-occupiers and renters γ. These targets put restriction on the parameters related to the exit rate from the city, the arrival rate of match quality shock in the two markets, the distribution of credit cost, the bargaining power of the seller and the discount rate.

Finally, we choose the elasticity of the owner-occupier's moving rate with respect to the tax increase, β, to match the decrease in transactions observed in the data.

5.2 Quantitative effects of transaction taxes

The steady state effects of the rise in transaction taxes $\tau_h = \tau_k$ from 1.08% to 2.04% are reported in Table 5. The model is calibrated to match the 10% decline in transactions. The model implies that tax revenue increased by 52% despite the tax rates increased by 89% reflecting the drop in the sales transactions. The model predicts a decline in mobility within the ownership market where an average homeowner now lives in a property for 8.4% longer. The price-rent ratio declines by 1.2%, and the time on the market goes up by 6.8%. The decrease in price is 1.4% which is larger than the 0.96% tax change itself, which follows from the anticipation effect of paying again with subsequent

Table 4. *Calibration of the parameters*

Parameter description	Notation	Value
Households per property	ϕ	1
Exit rate from city	ρ	0.0714
Discount rate	r	0.033
Property maintenance cost	M	0.177
Extra maintenance/management cost in rental market	M_l	0.037
Minimum new match utility in ownership market	ζ_o	1
Minimum new match utility in rental market	ζ_l	0.734
Shape parameter of new match quality in ownership market	λ_o	27.3
Shape parameter of new match quality in rental market	λ_l	41.6
Arrival rate of match quality shock in ownership market	a_o	0.147
Arrival rate of match quality shock in rental market	a_l	0.257
Match quality shock in ownership market	δ_o	0.875
Probability of drawing new credit cost	ς	0.405
Mean of credit cost draw	μ	1.60
Standard deviation of credit cost draw	σ	1.39
Transaction costs of buyers excluding taxes	$C_k = C_h$	0
Transaction costs of sellers	C_u	0.307
Transaction costs of landlords	C_l	0.038
Transaction costs of renters	C_w	0.038
Flow search costs of investors	$F_k = F_h$	0.271
Flow search costs of renters	F_w	0.256
Meeting efficiency in ownership market	A_o	135.3
Meeting efficiency in rental market	A_l	193.5
Elasticity of ownership market meetings w.r.t. sellers	η_o	0.253
Elasticity of rental market meetings w.r.t. landlords	η_l	0.50
Bargaining power of seller in ownership market	$\omega_o = \omega_k$	0.253
Bargaining power of landlord in rental market	ω_l	0.50

Notes: See appendix A.4 for data sources and appendix A.5 for the calibration procedure.

moves. The homeownership rate declines by 1.7% and the number of leases goes up by 1.9%.

Intuitively, households respond to an increase in transaction tax in three ways. First, home buyers become pickier ex-ante. This is because match quality is persistent, so homebuyers choose to start with a higher match quality to reduce future incidences of move. This results in longer time-to-sell. Second, once matched, a higher tax increases the cost of moving, so owner-occupiers become more tolerant towards lower match quality. This results in longer time-to-move. Finally, higher taxes reduce the joint surplus in the ownership market as part of the surplus was extracted by the taxes. This lowers the incentive for a renter to enter the ownership market. This increases the demand for rental properties and attracts more entry of investors.

Table 5. *Simulations of the model following the increase in land transfer tax*

Variable	Steady-state effect
Ownership market transactions (sales)	−10% (target)
Leasing transactions	1.9%
Homeownership rate	−1.6% (−0.8 p.p.)
Rent-to-own flow	−2.4%
Time-to-move for home-owners	8.4%
Time on ownership market	6.8%
Price paid by home-buyers	−1.4%
Price-to-rent ratio	−1.1%
Tax revenue	52%
Tax rate	89% (0.96 p.p.)

Notes: The solution procedure for the steady-state effects is described in appendix A.3.

The rise in time-to-sell and time-to-move both contribute to a fall in the transactions in the ownership market. The falling joint surplus in the ownership market together with the reduction in the effective share of surplus that goes to the seller, as shown in equation (19), implies a fall in price. The fall in prices translates to a fall in the price-to-rent ratio. The rise in the number of households in the rental market looking to rent provides an incentive for investors to enter, resulting in a lower homeownership rate and higher lease transactions.

Table 5 shows the steady state effects of the LTT increase. It is also possible to compute the transition dynamics using the procedure set out in appendix A.2. The results are shown in Figure 6.

5.3 Welfare analysis

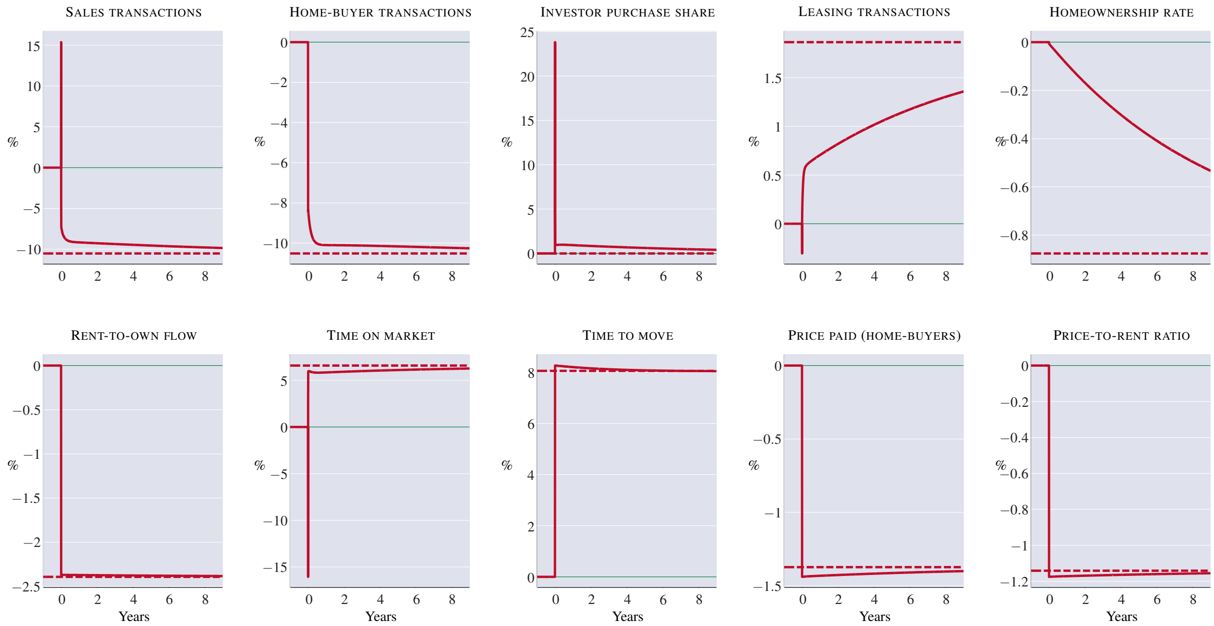
The flow value of steady-state utility net of costs averaged across all households is:

$$rW = h_o Q_o + h_l Q_l - M - h_l M_l - b_o F_h - b_l F_w - s_o u_o (C_h + C_u) - s_l u_l (C_w + C_l) + (\rho\phi + a_l \varsigma h_l) G_m(z)\bar{z}, \quad (67)$$

where Q_o is average match quality across homeowners:

$$Q_o = \frac{\lambda_o}{\lambda_o - 1}\left(\frac{n_o + \rho}{a_o + \rho} y_o + \frac{a_o - n_o}{a_o + \rho} x_o\right).$$

Figure 6: *Dynamic response to LTT change*



This is multiplied by the number of homeowners h_o who are not looking to move. Average match quality for renters is

$$Q_l = \frac{\lambda_l}{\lambda_l - 1} y_l \,,$$

which is multiplied by h_l in the expression for welfare. Maintenance costs for the unit measure of properties must be paid irrespective of housing tenure, and additional costs M_l are paid for the measure h_l of renters. Those households who are searching at a particular point in time incur search costs $b_o F_h + b_l F_w$ in total. Transactions occur at rate $s_o u_o$ in the ownership market and incur transaction costs $C_h + C_u$ each. Note that tax revenue from the land transfer tax is not deducted from welfare as this is presumed to be spent on public goods from which households derive an equal amount of utility. Transactions occur at rate $s_l u_l$ in the rental market and incur transaction costs $C_w + C_l$ each. Finally, there is a measure $(\rho\phi + a_l \varsigma h_l) G_m(z)$ of first-time buyers, who incur average credit cost $\bar{z}$ (modelled as a one-off cost of becoming a homeowner).

The welfare costs of the LTT are substantial. Per dollar of tax revenue raised, the LTT generates a welfare loss equivalent to 45% of the increase in tax revenue. The welfare loss is due to distortions within and across the two housing markets, with each accounting for about half of the loss. Within the ownership market, the loss is mainly due to the drop in match quality. The loss is partly offset by the reduction in search costs and non-tax transaction costs that are saved due to less moving taking place. The loss is much smaller within the rental market due to the small rise in rental activities. The source of welfare loss in the rental market is due to search and transaction costs. The welfare

Table 6. *Welfare effects of increase in land transfer tax predicted by model*

Variable	Result
Increase in deadweight loss relative to increase in tax revenue	45%
Percentage of welfare loss attributable to within-market effects	53%
Percentage of welfare loss attributable to across-market effects	47%

loss across the two market is due to the drop in homeownership rate, given average match quality is higher for owner-occupiers than renter-occupiers, and the increase in rental management. This loss is partly offset by the reduction in credit costs to become a homeowner.

The welfare loss we found is significantly larger than the 12.5% found by Dachis, Duranton and Turner (2012) for the Toronto LTT. This large welfare loss we found are largely accounted for by considering the mobility within the ownership market and across the two markets. The fall in mobility within the ownership market implies a fall in the average match quality whereas the fall in rent-to-own flow implies a fall in homeownership rate.

6 Conclusions

Using a unique dataset on housing sales and leasing transactions, this paper documents two novel effects of a higher transaction tax. First, there is a rise in buy-to-let transactions and a fall in owner-occupier transactions despite the same tax applying to both. Second, there is a simultaneous fall in the price-to-rent ratio and in the sales-to-leases ratio. Both effects operate through the extensive margin across the ownership and rental market.

The paper builds a tractable model where households choose renting or owning where entry to the ownership market requires paying a cost of accessing credit. The higher transaction tax distorts allocation of properties across the two markets by reducing the homeownership rate and distorts the allocation within the ownership market by reducing mobility. The calibrated model implies a substantial welfare loss equivalent to 45% of the increase in tax revenue, with half due to the misallocation across the rental and ownership markets.

References

ANENBERG, E. AND BAYER, P. (2020), "Endogenous sources of volatility in housing markets: The joint buyer-seller problem", *International Economic Review*, **61(3)**:1195–1228 (Aug.). 4

Bachmann, R. and Cooper, D. (2014), "The ins and arounds in the U.S. housing market", working paper 14–3, Federal Reserve Bank of Boston. 3

Benjamin, J. D., Coulson, N. E. and Yang, S. X. (1993), "Real estate transfer taxes and property values: The Philadelphia story", *Journal of Real Estate Finance and Economics*, **7(2)**:151–157. 1

Besley, T., Meads, N. and Surico, P. (2014), "The incidence of transaction taxes: Evidence from a stamp duty holiday", *Journal of Public Economics*, **119**:61–70 (Nov.). 1, 4

Best, M. and Kleven, H. (2018), "Housing market responses to transaction taxes: Evidence from notches and stimulus in the U.K.", *Review of Economic Studies*, **85(1)**:157–193 (Jan.). 1, 4, 11

Bracke, P. (2015), "House prices and rents: Microevidence from a matched data set in central London", *Real Estate Economics*, **43(2)**:403–431. 6

Chambers, M., Garriga, C. and Schlagenhauf, D. E. (2009), "Accounting for changes in the homeownership rate", *International Economic Review*, **50(3)**:677–726 (Aug.). 4

Cho, Y., Li, S. M. and Uren, L. (2021), "Stamping out stamp duty: Property or consumption taxes?", working paper 1/2021, Centre for Applied Macroeconomic Analysis. 5

Dachis, B., Duranton, G. and Turner, M. A. (2012), "The effects of land transfer taxes on real estate markets: evidence from a natural experiment in Toronto", *Journal of Economic Geography*, **12(2)**:327–354 (Mar.). 1, 3, 4, 5, 9, 11, 30, 35, 39

Davidoff, I. and Leigh, A. (2013), "How do stamp duties affect the housing market?", *Economic Record*, **89(286)**:396–410 (Sep.). 1, 4

Díaz, A. and Jerez, B. (2013), "House prices, sales, and time on the market: A search-theoretic framework", *International Economic Review*, **54**:837–872 (Aug.). 4

Eerola, E., Harjunen, O., Lyytikäinen, T. and Saarimaa, T. (2019), "Effects of housing transfer taxes on household mobility", working paper 7750, CESifo. 1, 4

Fisher, J. D. M. and Gervais, M. (2011), "Why has home ownership fallen among the young?", *International Economic Review*, **52(3)**:883–912 (Aug.). 4

Floetotto, M., Kirker, M. and Stroebel, J. (2016), "Government intervention in the housing market: Who wins, who loses?", *Journal of Monetary Economics*, **80**:106–123 (Jun.). 4

Fritzsche, C. and Vandrei, L. (2019), "The german real estate transfer tax: Evidence for single-family home transactions", *Regional Science and Urban Economics*, **74**:131–143. 1, 4, 11

Gabrovski, M. and Ortego-Marti, V. (2019), "The cyclical behavior of the Beveridge curve in the housing market", *Journal of Economic Theory*, **181**:361–381 (May). 4

Genesove, D. and Han, L. (2012), "Search and matching in the housing market", *Journal of Urban Economics*, **72**:31–45 (July). 4, 48

Glaeser, E. L. and Gyourko, J. (2007), "Arbitrage in housing markets", working paper 13704, NBER. 3, 6

Goodman, L. S. and Mayer, C. (2018), "Homeownership and the American Dream", *Journal of Economic Perspectives*, **32(1)**:31–58. 4

Guren, A. (2018), "House price momentum and strategic complementarity", *Journal of Political Economy*, **126(3)**:1172–1218 (Jun.). 4

Halket, J., Pignatti, M. and di Custoza, M. (2015), "Homeownership and the scarcity of rentals", *Journal of Monetary Economics*, **76**:107–123 (Nov.). 4

Han, L. and Strange, W. C. (2015), "The microstructure of housing markets: Search, bargaining, and brokerage", in G. Duranton, J. V. Henderson and W. C. Strange (eds.), *Handbook of Regional And Urban Economics*, vol. 5, North Holland, chap. 13, pp. 813–886. 4

Head, A., Lloyd-Ellis, H. and Sun, H. (2014), "Search, liquidity, and the dynamics of house prices and construction", *American Economic Review*, **104**:1172–1210 (Apr.). 4

Henry, K., Harmer, J., Piggott, J., Ridout, H. and Smith, G. (2009), "Australia's future tax system", Final report, Commonwealth of Australia. 3

Hilber, C. A. L. and Lyytikäinen, T. (2017), "Transfer taxes and household mobility: Distortion on the housing or labor market?", *Journal of Urban Economics*, **101**:57–73 (Sep.). 1, 4, 13

Ioannides, Y. M. and Zabel, J. E. (2017), "Vacancies in housing and labor markets", working paper, Tufts University. 4

Kaas, L., Kocharkov, G., Preugschat, E. and Siassi, N. (2021), "Low homeownership in germany?a quantitative exploration", *Journal of the European Economic Association*, **19**:128–164. 5

Kopczuk, W. and Munroe, D. (2015), "Mansion tax: The effect of transfer taxes on the residential real estate market", *American Economic Journal: Economic Policy*, **7(2)**:214–257 (May). 1, 4

Lenoel, C., Matsu, J. and Naisbitt, B. (2018), "International evidence review on housing taxation", Final report, UK Collaborative Centre for Housing Evidence. 3

Lundborg, P. and Skedinger, P. (1999), "Transaction taxes in a search model of the housing market", *Journal of Urban Economics*, **45(2)**:385–399 (Mar.). 4

Määttänen, N. and Terviö, M. (2020), "Welfare effects of housing transaction taxes", discussion paper 12551, Centre for Economic Policy Research. 1, 4

Mirrlees, J., Adam, S., Besley, T., Blundell, R., Bond, S., Chote, R., Gammie, M., Johnson, P., Myles, G. and Poterba, J. (2010), *Dimensions of Tax Design*, Oxford University Press. 3

Moen, E. R., Nenov, P. T. and Sniekers, F. (2021), "Buying first or selling first in housing markets", *Journal of the European Economic Association*, forthcoming. 4

Ngai, L. R. and Sheedy, K. D. (2020), "The decision to move house and aggregate housing-market dynamics", *Journal of the European Economic Association*, **18(5)**:2487–2531 (Oct.). 4

Ngai, L. R. and Tenreyro, S. (2014), "Hot and cold seasons in the housing market", *American Economic Review*, **104(12)**:3991–4026 (Dec.). 4

Piazzesi, M., Schneider, M. and Stroebel, J. (2020), "Segmented housing search", *American Economic Review*, **110(3)**:720–59 (Mar.). 4

Poterba, J. M. (1992), "Taxation and housing: Old questions, new answers", *American Economic Review*, **82(2)**:237–242 (May). 4

Rutherford, J., Rutherford, R. and Yavas, A. (2021), "Micro evidence relating to house rents, prices investors from a matched dataset", Working paper, European Central Bank. 6

SCANLON, K., WHITEHEAD, C. AND BLANC, F. (2017), "A taxing question: Is stamp duty land tax suffocating the English housing market?", Report, LSE and Family Building Society. 3

SHILLER, R. J. (2007), "Understanding recent trends in house prices and home ownership", working paper 13553, NBER. 6

SLEMROD, J., WEBER, C. AND SHAN, H. (2017), "The behavioral response to housing transfer taxes: Evidence from a notched change in D.C. policy", *Journal of Urban Economics*, **100**:137–153 (Jul.). 1

SOMMER, K. AND SULLIVAN, P. (2018), "Implications of US tax policy for house prices, rents, and homeownership", *American Economic Review*, **108(2)**:241–274 (Feb.). 4

VAN OMMEREN, J. AND VAN LEUVENSTEIJN, M. (2005), "New evidence of the effect of transaction costs on residential mobility", *Journal of Regional Science*, **45(4)**:681–702 (Nov.). 1, 4, 13

WHEATON, W. C. (1990), "Vacancy, search, and prices in a housing market matching model", *Journal of Political Economy*, **98(6)**:1270–1292 (Dec.). 4

A Appendices

A.1 Data appendix

Table 7. *Land transfer taxes in Toronto*

City of Toronto LTT Tax Rate by Value ($) (Effective 1 February 2008)		Province of Ontario LTT Tax Rate by Value ($) (Effective 7 May 1997)	
0-55,000	0.5%	0-55,000	0.5%
55,000-400,000	1.0%	55,000-250,000	1.0%
400,000+	2.0%	250,000-400,000	1.5%
		400,000+	2.0%

Sources: Municipal Land Transfer Tax, City of Toronto, http://www.toronto.ca/taxes/mltt.htm; Provincial Land Transfer Tax, Historical Land Transfer Tax Rates, Province of Ontario. Reproduced from Dachis, Duranton and Turner (2012).
Note: For the Municipal LTT, exemptions are given to first-time homebuyers for the value of a purchase under $400,000 and for the provincial LTT exemptions are given to first-time home buyers for the value of a purchase under $227,500.

Table 8. *Summary statistics, Greater Toronto Area, 2006-2010*

	Mean	St Dev	Obs
Total Sales	5.65	8.66	51,164
Investor Purchases	0.18	0.88	51,164
Owner Purchases	5.46	8.06	51,164
Leases	1.18	6.65	51,164
Leases/Sales	0.13	0.27	50,621
Avg Time to Move	979	391	19,023
Avg Time on Market	32.70	26.17	51,164
Avg Sales Price	370,911	222,458	51,164
Avg Owner Purchase Price	371,440	224,037	50,890
Avg Investor Purchase Price	368,708	280,427	5,625
Avg Rent	1,698	795	16,863
Avg Price to Average Rent	19.76	8.69	51,164
Avg Price Rent Ratio on Same Property	15.68	5.90	4,026

The estimates sample includes 51,164 unique market segments defined by Year $\times$Month $\times$ HouseType $\times$ Community. The sample contains 248 residential communities with five distinct property types: Detached, Semi-Detached, Row Townhouse, Condominium Townhouse, Condominium Apartment. The LTT was implemented in February 2008 and the sample covers transactions between January 2006 and February 2010.

A.2 Derivation of model equations

A.2.1 Value functions and thresholds for the ownership market

The value function $H(\varepsilon)$ is increasing in ε. Assuming $\delta_o y_o < x_o$ at all times, by taking ε in a neighbourhood above y_o or any value below, the Bellman equation (5) reduces to the following because $H(\delta_o\varepsilon) < B_o + U_o$:

$$rH(\varepsilon) = \varepsilon - M + a_o(B_o + U_o - H(\varepsilon)) + \rho(U_o - H(\varepsilon)) + \dot{H}(\varepsilon)\,.$$

This simplifies to:

$$(r+\rho+a_o)H(\varepsilon) - \dot{H}(\varepsilon) = \varepsilon - M + a_o B_o + (\rho + a_o)U_o\,, \tag{A.1}$$

and by differentiating both sides with respect to ε in the restricted range:

$$(r+\rho+a_o)H'(\varepsilon) - \dot{H}'(\varepsilon) = 1\,.$$

For a given ε, this specifies a first-order differential equation in time for $H'(\varepsilon)$. Since $H'(\varepsilon)$ is not a state variable, there exists a unique stable solution $H'(\varepsilon) = 1/(r+\rho+a_o)$, which is constant over time ($\dot{H}'(\varepsilon) = 0$). As $H'(\varepsilon)$ is independent of ε, integration over match quality ε shows the value function $H(\varepsilon)$ has the form

$$H(\varepsilon) = \underline{H} + \frac{\varepsilon}{r+\rho+a_o}\,, \quad \text{with} \quad \dot{H}(\varepsilon) = \dot{\underline{H}}\,, \tag{A.2}$$

where $\underline{H}$ is independent of ε, but may be time varying. This result is valid for ε in a neighbourhood above y_o and all values below. Substitution back into (A.1) shows that $\underline{H}$ satisfies the differential equation:

$$(r+\rho+a_o)\underline{H} - \dot{\underline{H}} = a_o B_o + (\rho+a_o)U_o - M\,. \tag{A.3}$$

Since $x_o < y_o$, equation (25) together with (A.2) imply that:

$$x_o = (r+\rho+a_o)(B_o + U_o - \underline{H})\,. \tag{A.4}$$

Equation (21) for the surplus and the definition of the transaction threshold (22) imply that y_o satisfies:

$$H(y_o) = H(x_o) + C_h + (1+\tau_h)C_u + \tau_h U_o\,, \tag{A.5}$$

and combining (A.2) with (A.5) yields:

$$y_o = x_o + (r+\rho+a_o)(C_h + (1+\tau_h)C_u + \tau_h U_o)\,. \tag{A.6}$$

The ownership market surplus $\Sigma_o(\varepsilon)$ is as given in equation (17) and is divided according to (19). The expected total surplus Σ_o is given by (23). The Bellman equation for a buyer (3) can therefore be expressed as the following differential equation:

$$(r+\rho)B_o - \dot{B}_o = (1-\omega_o^*)q_o\Sigma_o - F_h\,. \tag{A.7}$$

The total surplus in trade with an investor and the division of that surplus are as given in equations (37) and (39). Together with the homeowner surplus, the Bellman equation of a seller (4) is the differential equation:

$$rU_o - \dot{U}_o = \theta_o q_o\left(\omega_o^*(1-\xi)\Sigma_o + \omega_k^*\xi\Sigma_k\right) - M\,. \tag{A.8}$$

Using equations (13), (21), and (22), the expected homeowner surplus in (23) can be written as:

$$\Sigma_o = \int_{y_o}^{\infty} \lambda_o \zeta_o^{\lambda_o} \varepsilon^{-(\lambda_o+1)}\Sigma_o(\varepsilon)\mathrm{d}\varepsilon = \int_{y_o}^{\infty} \frac{\lambda_o \zeta_o^{\lambda_o}\varepsilon^{-(\lambda_o+1)}(H(\varepsilon) - H(y_o))}{1+\tau_h\omega_o^*}\mathrm{d}\varepsilon\,. \tag{A.9}$$

Make the following definition of $\bar{H}(\varepsilon)$ for an arbitrary level of match quality ε, and note the link with Σ_o:

$$\bar{H}(\varepsilon) = \int_{\varepsilon'=\varepsilon}^{\infty} \lambda_o \varepsilon^{\lambda_o}\varepsilon'^{-(\lambda_o+1)}(H(\varepsilon') - H(\varepsilon))\mathrm{d}\varepsilon'\,, \quad \text{where } \Sigma_o = \frac{\zeta_o^{\lambda_o} y_o^{-\lambda_o}\bar{H}(y_o)}{1+\tau_h\omega_o^*}\,. \tag{A.10}$$

Now restrict attention to ε such that $\delta_o\varepsilon < x_o$, so (5) implies $rH(\varepsilon) = \varepsilon - M + a_o(B_o + U_o - H(\varepsilon)) + \rho(U_o -$

$H(\varepsilon)) + \dot{H}(\varepsilon)$. Since $\delta_o y_o < x_o$, this limits ε to a neighbourhood above y_o and all values below. Using (25):

$$r(H(\varepsilon') - H(\varepsilon)) = (\varepsilon' - \varepsilon) + a_o\left(\max\{H(\delta_o\varepsilon'), H(x_o)\} - H(\varepsilon')\right) - a_o(H(x_o) - H(\varepsilon)) \\ - \rho(H(\varepsilon') - H(\varepsilon)) + (\dot{H}(\varepsilon') - \dot{H}(\varepsilon)),$$

which holds for any $\varepsilon' \geq \varepsilon$. This simplifies to:

$$(r + \rho + a_o)(H(\varepsilon') - H(\varepsilon)) - (\dot{H}(\varepsilon') - \dot{H}(\varepsilon)) = (\varepsilon' - \varepsilon) + a_o \max\{H(\delta_o\varepsilon') - H(x_o), 0\},$$

and multiplying both sides by $\lambda_o \varepsilon^{\lambda_o} \varepsilon'^{-(\lambda_o+1)}$, integrating over ε', and using the definition of $\bar{H}(\varepsilon)$ in (A.10):

$$(r + \rho + a_o)\bar{H}(\varepsilon) - \dot{\bar{H}}(\varepsilon) = \int_{\varepsilon'=\varepsilon}^{\infty} \lambda_o \varepsilon^{\lambda_o} \varepsilon'^{-(\lambda_o+1)} \left((\varepsilon' - \varepsilon) + a_o \max\{H(\delta_o\varepsilon') - H(x_o), 0\}\right) \mathrm{d}\varepsilon', \quad \text{(A.11)}$$

where the time derivative of $\bar{H}(\varepsilon)$ is obtained from (A.10):

$$\dot{\bar{H}}(\varepsilon) = \int_{\varepsilon'=\varepsilon}^{\infty} \lambda_o \varepsilon^{\lambda_o} \varepsilon'^{-(\lambda_o+1)} (\dot{H}(\varepsilon') - \dot{H}(\varepsilon)) \mathrm{d}\varepsilon'.$$

In (A.11), the term in $(\varepsilon' - \varepsilon)$ integrates to $\varepsilon/(\lambda_o - 1)$ using the formula for the mean of a Pareto distribution. The second term is zero for $\varepsilon' < x_o/\delta_o$ because $H(\delta_o\varepsilon')$ is increasing in ε'. Hence, equation (A.11) becomes:

$$(r + \rho + a_o)\bar{H}(\varepsilon) - \dot{\bar{H}}(\varepsilon) = \frac{\varepsilon}{\lambda_o - 1} + a_o \varepsilon^{\lambda_o} \int_{\varepsilon'=x_o/\delta_o}^{\infty} \lambda_o \varepsilon'^{-(\lambda_o+1)} (H(\delta_o\varepsilon') - H(x_o)) \mathrm{d}\varepsilon',$$

and with the change of variable $\varepsilon'' = \delta_o\varepsilon'$ in the second integral, this can be written as:

$$(r + \rho + a_o)\bar{H}(\varepsilon) - \dot{\bar{H}}(\varepsilon) = \frac{\varepsilon}{\lambda_o - 1} + a_o \delta_o^{\lambda_o} \varepsilon^{\lambda_o} \int_{\varepsilon''=x_o}^{\infty} \lambda_o \varepsilon''^{-(\lambda_o+1)} (H(\varepsilon'') - H(x_o)) \mathrm{d}\varepsilon''. \quad \text{(A.12)}$$

Make the following definition of a new variable X_o:

$$X_o(t) = (\lambda_o - 1)\left(r + \rho + a_o(1 - \delta_o^{\lambda_p})\right) \int_{t'=t}^{\infty} (r + \rho + a_o) e^{-(r+\rho+a_o)(t'-t)} \Bigg(\\ \int_{\varepsilon'=x_o}^{\infty} \lambda_o \varepsilon'^{-(\lambda_o+1)} (H(\varepsilon', t') - H(x_o, t')) \mathrm{d}\varepsilon' \Bigg) \mathrm{d}t', \quad \text{(A.13)}$$

and by differentiating with respect to time t this variable must satisfy the differential equation:

$$(r + \rho + a_o)X_o - \dot{X}_o = (\lambda_o - 1)(r + \rho + a_o)\left(r + \rho + a_o(1 - \delta_o^{\lambda_p})\right) x_o^{-\lambda_o} \bar{H}(x_o) \\ = (\lambda_o - 1)(r + \rho + a_o)\left(r + \rho + a_o(1 - \delta_o^{\lambda_p})\right) \int_{\varepsilon'=x_o}^{\infty} \lambda_o \varepsilon'^{-(\lambda_o+1)} (H(\varepsilon') - H(x_o)) \mathrm{d}\varepsilon'. \quad \text{(A.14)}$$

which uses the definition of $\bar{H}(\varepsilon)$ in (A.10). Substituting into equation (A.12):

$$(r + \rho + a_o)\bar{H}(\varepsilon) - \dot{\bar{H}}(\varepsilon) = \frac{1}{\lambda_o - 1}\left(\varepsilon + \frac{a_o \delta_o^{\lambda_o} \varepsilon^{\lambda_o} \left((r + \rho + a_o)X_o - \dot{X}_o\right)}{(r + \rho + a_o)\left(r + \rho + a_o(1 - \delta_o^{\lambda_p})\right)}\right),$$

and by collecting terms this can be written as:

$$(r + \rho + a_o)\left(\bar{H}(\varepsilon) - \frac{a_o \delta_o^{\lambda_o} \varepsilon^{\lambda_o}}{(\lambda_o - 1)(r + \rho + a_o)\left(r + \rho + a_o(1 - \delta_o^{\lambda_p})\right)} X_o\right) \\ - \left(\dot{\bar{H}}(\varepsilon) - \frac{a_o \delta_o^{\lambda_o} \varepsilon^{\lambda_o}}{(\lambda_o - 1)(r + \rho + a_o)\left(r + \rho + a_o(1 - \delta_o^{\lambda_p})\right)} \dot{X}_o\right) = \frac{\varepsilon}{\lambda_o - 1}.$$

Since the right-hand side is time invariant and none of the variables are predetermined, it follows for each fixed ε there is a unique stable solution for $\bar{H}(\varepsilon) - a_o \delta_o^{\lambda_o} \varepsilon^{\lambda_o} X_o/((\lambda_o - 1)(r + \rho + a_o)(r + \rho + a_o(1 - \delta_o^{\lambda_p})))$ that is time-invariant and equal to $\varepsilon/((\lambda_o - 1)(r + \rho + a_o))$. This demonstrates that for any given ε in a neighbourhood above y_o or any value below it, the function $\bar{H}(\varepsilon)$ is given by:

$$\bar{H}(\varepsilon) = \frac{1}{(\lambda_o - 1)(r + \rho + a_o)} \left(\varepsilon + \frac{a_o \delta_o^{\lambda_o} \varepsilon^{\lambda_o}}{r + \rho + a_o(1 - \delta_o^{\lambda_p})} X_o \right) . \tag{A.15}$$

Evaluating (A.15) at $\varepsilon = x_o$ and multiplying by $(\lambda_o - 1)(r + \rho + a_o)(r + \rho + a_o(1 - \delta_o^{\lambda_p}))x_o^{-\lambda_o}$:

$$(\lambda_o - 1)(r + \rho + a_o) \left(r + \rho + a_o(1 - \delta_o^{\lambda_p}) \right) x_o^{-\lambda_o} \bar{H}(x_o) = \left(r + \rho + a_o(1 - \delta_o^{\lambda_p}) \right) x_o^{1-\lambda_o} + a_o \delta_o^{\lambda_o} X_o ,$$

and substituting into (A.14) shows that X_o satisfies the differential equation in the moving threshold x_o:

$$\left(r + \rho + a_o(1 - \delta_o^{\lambda_p}) \right) X_o - \dot{X}_o = \left(r + \rho + a_o(1 - \delta_o^{\lambda_p}) \right) x_o^{1-\lambda_o} . \tag{A.16}$$

Next, evaluating (A.15) at $\varepsilon = y_o$ and substituting into (A.10):

$$\Sigma_o = \frac{\zeta_o^{\lambda_o}}{(1 + \tau_h \omega_o^*)(\lambda_o - 1)(r + \rho + a_o)} \left(y_o^{1-\lambda_o} + \frac{a_o \delta_o^{\lambda_o}}{r + \rho + a_o(1 - \delta_o^{\lambda_o})} X_o \right) . \tag{A.17}$$

In summary, (A.3), (A.4), (A.6), (A.7), (A.8), (A.16), and (A.17) form a system of differential equations in y_o, x_o, X_o, Σ_o, $\underline{H}$, B_o, and U_o, which take as given Σ_k, q_o and ξ. The average level of prices follows from (24):

$$P = C_u + U_o + \frac{\omega_o^* \Sigma_o}{\pi_o} . \tag{A.18}$$

A.2.2 Laws of motion for the ownership market

The number of viewings v_{oo} done by home-buyers in the ownership market is:

$$v_{oo} = (1 - \xi)\theta_o q_o u_o , \quad \text{where} \quad q_o = A_o \theta_o^{-\eta_o} .$$

Sales and the sales rate to home-buyers are:

$$S_{oo} = \pi_o v_{oo} , \quad s_{oo} = S_{oo}/((1 - \xi)u_o) = \theta_o q_o \pi_o , \quad \text{where} \quad \pi_o = \zeta_o^{\lambda_o} y_o^{-\lambda_o} .$$

with π_o denoting the acceptance probability for a home-buyer conditional on a viewing. Viewings, sales, and the sales rate for investors are:

$$v_k = \xi \theta_o q_o u_o , \quad S_k = v_k , \quad \text{and} \quad s_k = S_k/(\xi u_o) = \theta_o q_o .$$

Total sales and the average sales rate are:

$$S_o = S_{oo} + S_k , \quad \text{and} \quad s_o = (1 - \xi)s_{oo} + \xi s_k .$$

Let N_o denote moves within the city for homeowners and n_o the moving rate (excluding exit from city at rate ρ). The law of motion for u_o is:

$$\dot{u}_o = N_o + \rho h_o - S_o = (n_o + \rho)h_o - s_o u_o , \quad \text{where} \quad n_o = \frac{N_o}{h_o} .$$

Now consider moving N_o. The group of existing homeowners h_o is made up of matches that formed at various points in the past and have survived to the present. Moving requires that homeowners receive an idiosyncratic shock, which has arrival rate a_o independent of history. A measure $a_o h_o$ of households thus decide whether to move.

All matches began as a viewing with some initial match quality ε, which was drawn from a Pareto(ζ_o, λ_o) distribution (see 13). This match quality distribution has been truncated when transaction decisions were made and possibly when subsequent idiosyncratic shocks have occurred. Consider a group of surviving homeowners where initial match quality has been previously truncated at $\underline{\varepsilon}$. This group constitutes a fraction $\zeta_o^{\lambda_o} \underline{\varepsilon}^{-\lambda_o}$ of the initial measure of viewings, and the distribution of ε conditional on survival is Pareto$(\underline{\varepsilon}, \lambda_o)$. Among this group, consider those whose current match quality is a multiple Δ of original match quality ε, where Δ is equal to δ_o multiplied by the number of idiosyncratic shocks received.

Now consider a new idiosyncratic shock. Current match quality becomes $\varepsilon' = \delta_o \Delta \varepsilon$ in terms of initial match quality ε. Moving is optimal if $\varepsilon' < x_o$, so only those with initial match quality $\varepsilon \geq x_o/(\delta_o \Delta)$ survive. Since $\delta_o < 1$ and $\delta_o y_o < x_o$, there is a range of aggregate fluctuations that ensures $x_o/(\delta_o \Delta) > \underline{\varepsilon}$. Given the Pareto distribution, the proportion of the surviving group that does not move after the new shock is $\underline{\varepsilon}^{\lambda_o}(x_o/(\delta_o \Delta))^{-\lambda_o} = x_o^{-\lambda_o} \delta_o^{\lambda_o} \Delta^{\lambda_o} \underline{\varepsilon}^{\lambda_o}$. Since that surviving group is a fraction $\zeta_o^{\lambda_o} \underline{\varepsilon}^{-\lambda_o}$ of the original set of viewings, those that do not move after the new shock are a fraction $x_o^{-\lambda_o} \delta_o^{\lambda_o} \Delta^{\lambda_o} \underline{\varepsilon}^{\lambda_o} \times \zeta_o^{\lambda_o} \underline{\varepsilon}^{-\lambda_o} = (\zeta_o^{\lambda_o} x_o^{-\lambda_o} \delta_o^{\lambda_o}) \times \Delta^{\lambda_o}$ of that set of viewings. This is independent of any past truncation thresholds $\underline{\varepsilon}$ owing to the properties of the Pareto distribution.

The measure of the group choosing not to move after a new shock does depend on the total accumulated size Δ of past idiosyncratic shocks. Let Δ_o be the integral of Δ^{λ_o} over the measure of current and past viewings done by households who have not yet exited the city. Since the size of the group choosing not to move is a common multiple $\zeta_o^{\lambda_o} x_o^{-\lambda_o} \delta_o^{\lambda_o}$ of Δ^{λ_o}, the measure of those choosing not to move after a new shock is $a_o \zeta_o^{\lambda_o} x_o^{-\lambda_o} \delta_o^{\lambda_o} \Delta_o$. The size of the group of movers and the implied moving rate are therefore:

$$N_o = a_o h_o - a_o \zeta_o^{\lambda_o} x_o^{-\lambda_o} \delta_o^{\lambda_o} \Delta_o, \quad \text{and} \quad n_o = \frac{N_o}{h_o}. \tag{A.19}$$

Since the arrival of idiosyncratic shocks is independent of history, a fraction a_o of the group used to define Δ_o have Δ^{λ_o} reduced to $\delta_o^{\lambda_o} \Delta^{\lambda_o}$. Exit from the group occurs at rate ρ, and new viewings occur that start from $\Delta^{\lambda_o} = 1$ with measure v_{oo}. The differential equation for Δ_o is therefore:

$$\dot{\Delta}_o = v_{oo} + a_o(\delta_o^{\lambda_o} \Delta_o - \Delta_o) - \rho \Delta_o. \tag{A.20}$$

Define the following weighted average of current and past levels of home-buyer viewings v_{oo}:

$$\bar{v}_{oo}(t) = \int_{t' \to -\infty}^{t} (\rho + a_o(1 - \delta_o^{\lambda_o})) e^{-(\rho + a_o(1-\delta_o^{\lambda_o}))(t-t')} v_{oo}(t') \mathrm{d}t',$$

and note that it satisfies the differential equation:

$$\dot{\bar{v}}_{oo} + (\rho + a_o(1 - \delta_o^{\lambda_o})) \bar{v}_{oo} = (\rho + a_o(1 - \delta_o^{\lambda_o})) v_{oo}. \tag{A.21}$$

A comparison of (A.20) and (A.21) shows that $\Delta_o = \bar{v}_{oo}/(\rho + a_o(1 - \delta_o^{\lambda_o}))$, and substituting this into (A.19) yields an equation for the moving rate:

$$n_o = a_o - \frac{a_o \zeta_o^{\lambda_o} \delta_o^{\lambda_o} x_o^{-\lambda_o} \bar{v}_{oo}}{(\rho + a_o(1 - \delta_o^{\lambda_o})) h_o}. \tag{A.22}$$

A.2.3 Investors

The surplus from sales to investors is given by (41), so the equation for Σ_k is:

$$\Sigma_k = \frac{U_l - (1 + \tau_k) U_o - (1 + \tau_k) C_u - C_k}{1 + \omega_k^* \tau_k}. \tag{A.23}$$

Since free entry (10) implies $K = 0$ at all dates, then $\dot{K} = 0$ as well, and the Bellman equation (2) reduces to:

$$(1 - \omega_k^*) q_o \Sigma_k = F_k. \tag{A.24}$$

Taking U_l and U_o as given, equations (A.23) and (A.24) determine Σ_k and implicitly ξ.

A.2.4 Value functions and thresholds for the rental market

Adding together the Bellman equations (7) and (9) for the landlord and tenant value functions:

$$r(L(\varepsilon) + W(\varepsilon)) = \varepsilon - M - M_l + (\rho + a_l)(U_l - L(\varepsilon)) + a_l(1 - \varsigma)(B_l - W(\varepsilon)) \\ + a_l \varsigma (G_m(z)(B_o - \bar{z}) + (1 - G_m(z)) B_l - W(\varepsilon)) - \rho W(\varepsilon) + \dot{L}(\varepsilon) + \dot{W}(\varepsilon).$$

Letting $J(\varepsilon) = L(\varepsilon) + T(\varepsilon)$ denote the joint value, this can be rearranged and simplified, noting $B_o - B_l = z$:

$$(r+\rho+a_l)J(\varepsilon) = \varepsilon - M - M_l + (\rho+a_l)U_l + a_l B_l + a_l \varsigma G_m(z)(z-\bar{z}) + \dot{J}(\varepsilon)\,. \tag{A.25}$$

Differentiating with respect to ε:

$$(r+\rho+a_l)J'(\varepsilon) = 1 + \dot{J}'(\varepsilon)\,,$$

and this differential equation has a unique non-explosive solution for $J'(\varepsilon)$ for any given value of ε:

$$J'(\varepsilon) = \frac{1}{r+\rho+a_l}\,.$$

This time-invariant solution ($\dot{J}'(\varepsilon) = 0$) implies the solution for $J(\varepsilon)$ takes the following form:

$$J(\varepsilon) = \underline{J} + \frac{\varepsilon}{r+\rho+a_l}\,, \tag{A.26}$$

where $\underline{J}$ can be time varying in general. Substituting back into (A.25) and noting $\dot{J}(\varepsilon) = \dot{\underline{J}}$ shows that $\underline{J}$ satisfies the differential equation:

$$(r+\rho+a_l)\underline{J} = a_l B_l + (\rho+a_l)U_l - M - M_l + a_l \varsigma G_m(z)(z-\bar{z}) + \dot{\underline{J}}\,. \tag{A.27}$$

The total rental surplus from (30) implies that:

$$\Sigma_l(\varepsilon) = J(\varepsilon) - C_l - C_w - B_l - U_l\,. \tag{A.28}$$

Together with (A.26), the definition of the rental transaction threshold y_l in (33) implies:

$$y_l = (r+\rho+a_l)(B_l + U_l - \underline{J} + C_l + C_w)\,. \tag{A.29}$$

Using (33), (A.26), and (A.28), it follows that $\Sigma_l(\varepsilon) = (\varepsilon - y_l)/(r+\rho+a_l)$. The Pareto distribution in (13) then implies the expected rental surplus from (34) is:

$$\Sigma_l = \frac{\zeta_l^{\lambda_l} y_l^{1-\lambda_l}}{r+\rho+a_l}\,. \tag{A.30}$$

Using (29) and (31), the Bellman equation (6) for U_l becomes:

$$rU_l - \dot{U}_l = \omega_l \theta_l q_l \Sigma_l - M\,, \tag{A.31}$$

and the Bellman equation (8) for B_l becomes:

$$(r+\rho)B_l - \dot{B}_l = (1-\omega_l)q_l \Sigma_l - F_w\,. \tag{A.32}$$

The credit cost threshold z satisfies:

$$B_o - B_l = z\,. \tag{A.33}$$

In summary, equations (A.27), (A.29), (A.30), (A.31), (A.32), and (A.33) determine y_l, z, Σ_l, $\underline{J}$, B_l, and U_l.

The Bellman equation (7) can be written as follows:

$$(r+\rho+a_l)(L(\varepsilon) - U_l) = R(\varepsilon) - M - M_l - rU_l + \dot{L}(\varepsilon)\,,$$

and substituting from (A.31) implies that rents $R(\varepsilon)$ are:

$$R(\varepsilon) = M_l + \omega_l \theta_l q_l \Sigma_l + (r+\rho+a_l)(L(\varepsilon) - U_l) - (\dot{L}(\varepsilon) - \dot{U}_l)\,.$$

Since $S^l(\varepsilon) = L(\varepsilon) - U_l$ according to (26), which means $\dot{L}(\varepsilon) - \dot{U}_l = \dot{S}^l(\varepsilon)$, the division of the surplus $S^l(\varepsilon) = \omega_l S(\varepsilon)$ from (28) implies:

$$R(\varepsilon) = M_l + \omega_l \theta_l q_l \Sigma_l + \omega_l \left((r+\rho+a_l)S(\varepsilon) - \dot{S}(\varepsilon)\right)\,.$$

By substituting from the equation for the total rental market surplus in (30), the expression for rents becomes:

$$R(\varepsilon) = M_l + \omega_l(r+\rho+a_l)(C_w + C_l) + \omega_l \theta_l q_l \Sigma_l + \omega_l \left((r+\rho+a_l)\Sigma_l(\varepsilon) - \dot{\Sigma}_l(\varepsilon)\right)\,.$$

Noting that $\Sigma_l(\varepsilon) = -\dot{y}_l/(r+\rho+a_l)$ for all ε, and using the definition of average new rents R from (35):

$$R = M_l + \omega_l(r+\rho+a_l)(C_w+C_l) + \omega_l\theta_l q_l \Sigma_l + \omega_l\left((r+\rho+a_l)\frac{\Sigma_l}{\pi_l} + \frac{\dot{y}_l}{r+\rho+a_l}\right),$$

which can be written as:

$$R = M_l + \omega_l(r+\rho+a_l)(C_l+C_w) + \omega_l(r+\rho+a_l+\theta_l q_l \pi_l)\frac{\Sigma_l}{\pi_l} + \frac{\omega_l}{r+\rho+a_l}\dot{y}_l\,. \tag{A.34}$$

A.2.5 Market tightness

The total measure of properties in (43) and families in (44) together with the definition of market tightness $\theta_i = b_i/u_i$ imply:

$$(1-\xi)(\theta_o-1)u_o + (\theta_l-1)u_l = \phi - 1\,, \tag{A.35}$$

which gives a relationship between the market tightnesses θ_o and θ_l across the two markets for a given stock of properties for sale u_o and properties for rent u_l.

A.3 Existence of a steady state and solution method

Since the free entry condition condition must hold, equations (41) (42) imply that $U_l > U_o$, so the value of having a property for rent is larger than the value of having a property for sale. This means that landlords have no incentive to exit the market. Hence, there must be a steady state where there is no entry of new investors.

Equations for the steady state With no new entry of investors, $\xi = 0$, so in a steady state where $\dot{B}_o = 0$ and $\dot{U}_o = 0$, the Bellman equations (A.7) and (A.8) become:

$$(r+\rho)B_o = -F_h + (1-\omega_o^*)q_o\Sigma_o\,, \quad \text{and} \tag{A.36}$$

$$rU_o = -M + \omega_o^*\theta_o q_o \Sigma_o\,. \tag{A.37}$$

Substituting from (A.8) into (A.6):

$$y_o = x_o + (r+\rho+a_o)\left(C_h + C_u + \tau_h\left(C_u - \frac{M}{r} + \frac{\omega_o^*\theta_o q_o \Sigma_o}{r}\right)\right). \tag{A.38}$$

In a steady state with $\dot{\underline{H}} = 0$, (A.3) implies $(r+\rho+a_o)\underline{H} = a_o B_o + (\rho+a_o)U_o - M$. Substituting into (A.4) implies $x_o = (r+\rho+a_o)(B_o+U_o) - a_o B_o - (\rho+a_o)U_o + M$ and hence:

$$x_o = M + (r+\rho)B_o + rU_o\,,$$

and then substituting the values of B_o and U_o from (A.36) and (A.37):

$$x_o + F_h = (1-\omega_o^* + \omega_o^*\theta_o)\,q_o\Sigma_o\,. \tag{A.39}$$

With $\dot{X}_o = 0$ in steady state, equation (A.16) shows that $X_o = x_o^{1-\lambda_o}$. Substitution into (A.17) implies the expected surplus is:

$$\Sigma_o = \frac{\zeta_o^{\lambda_o}}{(r+\rho+a_o)(\lambda_o-1)(1+\tau_h\omega_o^*)}\left(y_o^{1-\lambda_o} + \frac{a_o\delta_o^{\lambda_o}x_o^{1-\lambda_o}}{r+\rho+a_o\left(1-\delta_o^{\lambda_o}\right)}\right). \tag{A.40}$$

The average transaction price P from (A.18) can be written as follows by using equation (A.37) for U_o:

$$P = \left(\frac{r+\theta_o q_o \pi_o}{r}\right)\left(\frac{\omega_o^*\Sigma_o}{\pi_o}\right) + C_u - \frac{M}{r}\,. \tag{A.41}$$

With $\dot{B}_l = 0$ and $\dot{U}_l = 0$, the Bellman equations (A.32) and (A.31) become:

$$rB_l = -F_w + (1-\omega_l)q_l\Sigma_l - \rho B_l\,, \tag{A.42}$$

$$rU_l = -M + \omega_l\theta_l q_l\Sigma_l\,. \tag{A.43}$$

In the steady state, $\dot{\underline{J}} = 0$, which yields $(r+\rho+a_l)\underline{J} = a_l B_l + (\rho+a_l)U_l - M - M_l + a_l\varsigma G_m(z)(z-\bar{z})$ using (A.27). Substituting into (A.29) implies:

$$y_l = M + M_l + (r+\rho)B_l + rU_l + (r+\rho+a_l)(C_l+C_w) - a_l\varsigma G_m(z)(z-\bar{z})\,,$$

and using (A.42) and (A.43) this becomes:

$$y_l = M_l - F_w + (r+a_l+\rho)(C_w+C_l) - a_l\varsigma G_m(z)(z-\bar{z}) + (1-\omega_l+\omega_l\theta_l)q_l\Sigma_l\,. \tag{A.44}$$

The rent equation (A.34) in steady state is:

$$R = M_l + \omega_l(r+\rho+a_l)(C_l+C_w) + \omega_l(r+\rho+a_l+\theta_l q_l\pi_l)\frac{\Sigma_l}{\pi_l}\,. \tag{A.45}$$

Substituting the values of Σ_k, U_o, and U_l from (A.37), (A.43), and (41) into (42) implies:

$$\omega_l\theta_l\frac{q_l}{r}\Sigma_l - (1+\tau_k)\omega_o^*\theta_o\frac{q_o}{r}\Sigma_o = C_k + (1+\tau_k)C_u - \tau_k\frac{M}{r} + \frac{(1+\tau_k\omega_k^*)F_k}{(1-\omega_k^*)q_o}\,. \tag{A.46}$$

By substituting B_o and B_l from (A.36) and (A.42) into (11):

$$(1-\omega_o^*)q_o\Sigma_o - (1-\omega_l)q_l\Sigma_l = (r+\rho)z + F_h - F_w\,. \tag{A.47}$$

The price paid by investors in equilibrium is obtained from (40), (42), and (A.37):

$$P_k = C_u - \frac{M}{r} + \omega_o^*\theta_o q_o\Sigma_o - \frac{\omega_k^* F_k}{(1-\omega_k^*)q_o}\,. \tag{A.48}$$

The procedure is to reduce the solution to a numerical search over market tightnesses θ_o and θ_l to find the roots of two equations that represent the equilibrium of the housing market.

Ownership-market transaction threshold Conditional on θ_o, within this search, there is also a numerical search to find the transaction threshold y_o in the ownership market. Taking a value of y_o, the moving threshold x_o must satisfy (A.38) and hence:

$$y_o = x_o + (r+\rho+a_o)\left(C_h + (1+\tau_h)C_u - \tau_h\frac{M}{r}\right) + \tau_h\left(\frac{r+\rho+a_o}{r}\right)\omega_o^*\theta_o q_o\Sigma_o\,.$$

Equation (A.39) implies $q_o\Sigma_o = (x_o+F_h)/(1-\omega_o^*+\omega_o^*\theta_o)$ and substituting this into the above and solving the linear equation for x_o:

$$x_o = \frac{y_o - (r+\rho+a_o)\left(C_h + (1+\tau_h)C_u + \tau_h\left(\frac{\omega_o^*\theta_o}{1-\omega_o^*+\omega_o^*\theta_o}\right)\frac{F_h}{r} - \tau_h\frac{M}{r}\right)}{1+\tau_h\left(\frac{\omega_o^*\theta_o}{1-\omega_o^*+\omega_o^*\theta_o}\right)\left(\frac{r+\rho+a_o}{r}\right)}\,. \tag{A.49}$$

Now combine equations (A.39) and (A.40) and use $q_o = A_o\theta_o^{-\eta_o}$ from (12):

$$x_o + F_h - \frac{(1-\omega_o^*+\omega_o^*\theta_o)A_o\theta_o^{-\eta_o}\zeta_o^{\lambda_o}}{(1+\tau_h\omega_o^*)(r+\rho+a_o)(\lambda_o-1)}\left(y_o^{1-\lambda_o} + \frac{a_o\delta_o^{\lambda_o}}{r+\rho+a_o(1-\delta_o^{\lambda_o})}x_o^{1-\lambda_o}\right) = 0\,. \tag{A.50}$$

Observe that the left-hand side of (A.50) is strictly increasing in x_o and y_o. As the value of x_o implied by (A.49) is strictly increasing in y_o, it follows that any solution of (A.49) and (A.50) for x_o and y_o is unique. Since the left-hand side of (A.50) is sure to be positive for large y_o, existence of a solution can be confirmed by checking whether the left-hand side is negative at $y_o = \zeta_o$, the minimum value of y_o.

Ownership-market variables Once y_o is found, the transaction probability in the ownership market conditional on a viewing is $\pi_o = (\zeta_o/y_o)^{\lambda_o}$. The sales rate is $s_o = \theta_o q_o\pi_o$. The moving threshold x_o is obtained from (A.49), and it can be verified whether $\delta_o y_o < x_o$ is satisfied. With x_o and y_o, the moving rate n_o is obtained from (62). Equation (A.39) implies the surplus is given by $\Sigma_o = (x_o + F_h)/((1-\omega_o^* + \omega_o^*\theta_o)q_o)$.

Rental-market variables Conditional on market-tightness θ_l, (12) implies $q_l = A_l\theta_l^{-\eta_l}$. The free-entry condition (A.46) can be written as follows, noting (18) implies $(1+\tau_k\omega_k^*)/(1-\omega_k^*) = 1/(1-\omega_k)$:

$$\omega_l\theta_l\frac{q_l}{r}\Sigma_l - (1+\tau_k)\omega_o^*\theta_o\frac{q_o}{r}\Sigma_o = C_k + (1+\tau_k)C_u - \tau_k\frac{M}{r} + \frac{F_k}{(1-\omega_k)q_o}. \tag{A.51}$$

Since q_o and Σ_o are known, the rental surplus Σ_l must be given by:

$$\Sigma_l = \frac{(1+\tau_k)\omega_o^*\theta_o q_o\Sigma_o + (C_k + (1+\tau_k)C_u)r - \tau_k M + \frac{r}{1-\omega_k}\frac{F_k}{q_o}}{\omega_l\theta_l q_l}. \tag{A.52}$$

With both Σ_o and Σ_l, the indifference condition (A.47) implies the credit cost z of the marginal homeowner:

$$z = \frac{(1-\omega_o^*)q_o\Sigma_o - (1-\omega_l)q_l\Sigma_l - F_h + F_w}{r+\rho}.$$

The fraction $G_m(z) = \Phi((\log z - \mu)/\sigma)$ of those drawing a credit cost less than z follows from (14), and the average level of credit costs $\bar{z}$ can be calculated from (15). With Σ_l, z, and $\bar{z}$ known, the transaction threshold y_l is derived from (A.44):

$$y_l = M_l - F_w + (r+\rho+a_l)(C_w + C_l) - a_l\varsigma G_m(z)(z-\bar{z}) + (1-\omega_l+\omega_l\theta_l)q_l\Sigma_l. \tag{A.53}$$

Checking that $y_l > \zeta_l$, the transaction probability conditional on a viewing $\pi_l = (\zeta_l/y_l)^{\lambda_l}$ follows from (13). The letting rate is thus $s_l = \theta_l q_l \pi_l$, and n_l is as given in (56).

Stocks and flows Equation (63) implies:

$$\left((q_l\pi_l+\rho)\frac{b_l}{u_l} - (a_l(1-\varsigma) + a_l\varsigma(1-G_m(z)))\frac{h_l}{u_l}\right)u_l = \rho\phi(1-G_m(z))$$

and using market tightness $\theta_l = b_l/u_l$ and the value of h_l/u_l implied by (58):

$$u_l = \frac{\rho\phi(1-G_m(z))}{(q_l\pi_l+\rho)\theta_l - \frac{s_l(a_l(1-\varsigma)+a_l\varsigma(1-G_m(z)))}{n_l+\rho}}. \tag{A.54}$$

After checking that $0 < u_l < 1$, the value of h_l is derived from (58) as $h_l = s_l u_l/(n_l+\rho)$, followed by a further check that $h_l + u_l < 1$. Equation (57) implies $h_o + u_o = u_o(s_o+n_o+\rho)/(n_o+\rho)$, and together with $h_o + u_o = 1-(h_l+u_l)$ from (43), it follows that:

$$u_o = \left(\frac{n_o+\rho}{s_o+n_o+\rho}\right)(1-(h_l+u_l)) \tag{A.55}$$

and $h_o = s_o u_o/(n_o+\rho)$.

Criteria for market-tightness solution Finally, two equations are needed to pin down the market tightnesses. Conditional on θ_o and θ_l, the values of u_o and u_l are obtained following the steps above and using (A.54) and (A.55). These must satisfy (A.35):

$$(\theta_o-1)u_o + (\theta_l-1)u_l + 1 - \phi = 0. \tag{A.56}$$

The second equation to check is (A.30) using the values of Σ_l and y_l derived above in (A.52) and (A.53) for given θ_o and θ_l:

$$\Sigma_l - \frac{\zeta_l^{\lambda_l}y_l^{1-\lambda_l}}{(\lambda_l-1)(r+\rho+a_l)} = 0. \tag{A.57}$$

Searching over values of θ_o and θ_l that satisfy (A.56) and (A.57), the equilibrium is found.

A.4 Calibration targets

The land transfer tax is the main transaction cost paid by buyers in the ownership market. The effective LTT rate, measured by the mean of land transfer taxes relative to sales prices across transactions, is 1.08% during the pre-policy period (January 2006 – January 2008), so $\tau_h = \tau_k = 0.011$. The parameters of the model are calibrated to match the pre-policy period in Toronto.

Transaction costs in the ownership market Excluding the land transfer tax, buyers in the ownership market may pay a home inspection cost of about \$500, which is very small relative to the average house price. So it is assumed that other than the LTT, buyers in the ownership market do not pay other transaction costs, so $C_k = C_h = 0$.

Turning to the seller side, the primary cost for the seller of a property is real estate agent commissions. Using Multiple Listing Service sales data, the average commission rate is about 4.5%. There are some other costs such as legal fees of around \$1,000, which are negligible, and some sellers may spend roughly \$2,500 on staging but in some cases the seller's agent covers this expense as part of their commission, so not all sellers pay for staging out of their own pocket. Thus, C_u is set to be 4.5% of the average house price.

Maintenance costs The maintenance cost as an owner, M, is set so that in equilibrium it is 2.6% of the average house price. This cost is made up of a 2% maintenance cost and a 0.06% property tax in Toronto. The extra maintenance cost of being a landlord, M_l, is set to be 8% of the average rent. This cost includes two parts: (1) approximately 5–7% that the landlord uses to hire a property manager; and (2) approximately 1% that the landlord uses to hire someone to take out garbage, shovel the snow, salt the walkways, etc.

Transaction costs in the rental market In Toronto, landlords typically pay one month's rent to real estate agents to lease their property. So C_l is set to equal $1/12$ of annual rent. Tenants in Toronto do not typically pay a monetary transaction cost when renting a property. This implies the fee T in the model is equal to zero, thus it follows from equation (32) that $C_w = ((1-\omega_l)/\omega_l)C_l$.

Flows in the housing markets The flows in the housing market are affected by time-to-move, time-to-sell, and viewings-per-sale. The information on time-to-move and time-to-sell is based on the Toronto MLS on sales and rental transactions during the pre-policy period. The average duration of stay before a homeowner moves out is 2,010 days, so $T_{mo} = 2010/365$. The average duration of stay for a tenant is 1,109 days, so $T_{ml} = 1109/365$. The time-to-sell for homebuyers is 24 days and the average time-to-rent is 18.7 days. During this period, the fraction of withdrawals out of total for-sale listings is 48% and that out of total for-lease listings is 22%. So we set $T_{so} = (24/365)/(1-0.48)$ and $T_{sl} = (18.7/365)/(1-0.22)$ in the model to incorporate the withdrawal rate in the data. This is because time-to-sell in the data is calculated from the final successful listing without accounting for earlier unsuccessful attempts, so the true time-to-sell is longer.

Using the 'Profile of Buyers and Sellers' survey collected by NAR, Genesove and Han (2012) report that for the period 2006–2009, the ratio of average time-to-buy to the the average time-to-sell is 1.28 and the average number of homes visited is 10.7. Thus, we set $T_{bo} = 1.28 \times T_{so}$ and $V_o = 10.7/(1-0.48)$ in the model to incorporate the withdrawal rate in the data. The idea is that viewings of properties that have been withdrawn from the market are not counted, so actual viewings are larger than the reported viewings. We do not have information on the number of homes that renters visit on average. According to an industry expert, renters visit fewer homes than buyers, so we assume it is half the viewings per sale by setting $V_l = V_o/2$. Note that the time taken for a household to find a rental property, T_{bl}, is not an independent target and it is pinned down by the steady-state condition linking the homeownership rate, time-to-move, and time-to-sell in the two markets.

Flow search costs There are no direct estimates of the flow costs of searching F_k, F_h, and F_w. The approach taken here is to base an estimate of search costs on the opportunity cost of the time spent searching. More specifically, for buyers in the ownership market, assume one property viewing entails the loss of a day's income, so the value of $F_k = F_h$ can be calibrated by adding up the costs of making the expected number of viewings. With viewings per sale equal to the average number of viewings made by a buyer, the total search cost should be equated to $0.5 \times V_o(Y/365)$, where Y denotes average annual income and assuming a viewing takes up half a day. Thus, the calibration sets $T_{bo}F_h = V_oY/365$, and by dividing both sides by PT_{bo}, this implies $F_h/P = 0.5 \times (1/365)(Y/P)(V_o/T_{bo})$. Taking the median household-level income from Stat Canada, we obtain a price-to-income ratio in Toronto of $P/Y = 5.6$ in 2007. Given the value of V_o/T_{bo}, the implied buyer's flow search cost, $F_h = F_k$ is 2.9% of the average house price.

Using the same logic for the flow search cost for tenants in the rental market, the ratio of flow search cost for a tenant relative to a homebuyer F_w/F_h is set to $0.5 \times (V_l/V_o) \times (T_{bo}/T_{bl})$, where it is assumed that viewing a rental properties takes half the time needed to view a property to buy.

Targets related to movement across the two housing markets Using the MLS data on sales and rental transactions in 2007, the price-to-rent ratio is 14.8 for the same property in Toronto. Based on the 2006 City of Toronto Profile Report, the homeownership rate is $h = 54\%$, the average age of homeowners is 53.3 and the average age of tenants is 45.0. Hence the difference between the average ages of homeowners and renters is $\gamma = 8.3$. There is no survey that specifically captures the proportion of first-time buyers. The Canadian Associated of Accredited Mortgage Professionals (now called Mortgage Professionals Canada) undertook a survey in 2015 and found that the fraction is as high as 45% of purchases, which is consistent with the 44% found in the 2018 Canadian Household Survey for the GTA area. On the other hand, the Canada Mortgage and Housing Corporation suggests that the fraction of first-time buyers is about one third. Based on this information, the calibration sets $\psi = 0.4$.

Mortgage costs The cost χ of becoming a homeowner is computed from a comparison of the mortgage rate r_m the household would face relative to the risk-free interest rate r_f on government bonds. The interest rates r_m and r_f are real interest rates. There is a spread between them due to unmodelled financial fricitions. The risk-free real rate r_f used to discount future cashflows need not be the same as the discount rate r applied to future utility from owning property (allowing for an unmodelled housing risk premium between r and r_f). All these interest rates are expected to remain constant over the mortgage term.

Suppose the household buys a property at price p at date $t = 0$ by taking out a mortgage with loan-to-value ratio ℓ. Assume the mortgage has term T and a constant real repayment α over its term. Let $D(t)$ denote the outstanding mortgage balance at date t, which has initial condition $D(0) = \ell p$ and terminal condition $D(T) = 0$. The mortgage balance evolves over time according to the differential equation:

$$\dot{D}(t) = r_m D(t) - \alpha \quad \text{and hence} \quad \frac{\mathrm{d}(e^{-r_m t}D(t))}{\mathrm{d}t} = -\alpha e^{-r_m t}.$$

Solving this differential equation using the initial condition $D(0) = \ell p$ implies:

$$D(t) = e^{r_m t}\ell p - \frac{\alpha}{r_m}(e^{r_m t} - 1).$$

The terminal condition $D(T) = 0$ requires that the constant real repayment α satisfies:

$$\alpha = \frac{r_m \ell p}{1 - e^{-r_m T}}.$$

Homeowners exit at rate ρ, in which case it is assumed they repay their mortgage in full (using the proceeds of selling their property). Hence, there is a probability $e^{-\rho t}$ that the date-t repayment α will be made, and a probability $\rho e^{-\rho t}$ that the whole balance $D(t)$ is repaid at date t. The credit cost χ is the present value of the expected stream of repayments discounted at rate r_f minus the amount borrowed (which would equal the present value of the repayments if $r_m = r_f$ in the absence of credit-market imperfections):

$$\chi = \int_{t=0}^{T} e^{-r_f t}e^{-\rho t}\alpha \mathrm{d}t + \int_{t=0}^{T} e^{-r_f t}e^{-\rho t}\rho D(t)\mathrm{d}t - \ell P.$$

To derive a formula for χ, first observe that

$$\int_{t=0}^{T} e^{-r_f t} e^{-\rho t} \mathrm{d}t = \frac{1-e^{-(r_f+\rho)T}}{r_f+\rho} \quad \text{and} \quad \int_{t=0}^{T} e^{-r_f t} e^{-\rho t} e^{r_m t} \mathrm{d}t = \frac{1-e^{-(r_f+\rho-r_m)T}}{r_f+\rho-r_m}.$$

Together with the formulas for $D(t)$ and α, the credit cost can thus be written as follows:

$$\chi = \frac{\left(\alpha + \frac{\rho\alpha}{r_m}\right)}{(r_f+\rho)}(1-e^{-(r_f+\rho)T}) + \frac{\rho\left(\ell p - \frac{\alpha}{r_m}\right)}{(r_f+\rho-r_m)}(1-e^{-(r_f+\rho-r_m)T}) - \ell p$$

$$= \left(\frac{(r_m+\rho)(1-e^{-(r_f+\rho)T})}{(r_f+\rho)(1-e^{-r_m T})} + \frac{\rho\left(1-\frac{1}{1-e^{-r_m T}}\right)(1-e^{-(r_f+\rho-r_m)T})}{(r_f+\rho-r_m)} - 1 \right) \ell p$$

$$= \frac{\left((r_m+\rho)(1-e^{-(r_f+\rho)T}) - \frac{\rho(r_f+\rho)}{r_f+\rho-r_m}(e^{-r_m T} - e^{-(r_f+\rho)T}) - (r_f+\rho)(1-e^{-r_m T})\right)\ell p}{(r_f+\rho)(1-e^{-r_m T})}$$

$$= \frac{\left((r_m-r_f) + \frac{\rho(r_f+\rho)-(r_m+\rho)(r_f+\rho-r_m)}{r_f+\rho-r_m} e^{-(r_f+\rho)T} - \frac{(r_f+\rho)(r_f+\rho-r_m)-\rho(r_f+\rho)}{r_f+\rho-r_m} e^{-r_m T}\right)\ell p}{(r_f+\rho)(1-e^{-r_m T})},$$

and dividing both sides by price p and simplifying:

$$\frac{\chi}{p} = \left(1 + \frac{r_m}{r_f+\rho-r_m} e^{-(r_f+\rho)T} - \frac{r_f+\rho}{r_f+\rho-r_m} e^{-r_m T}\right) \frac{(r_m-r_f)\ell}{(r_f+\rho)(1-e^{-r_m T})}.$$

This equation is used to determine calibration targets for the marginal credit cost z relative to the average property price P, and for the marginal credit cost z relative to the median credit cost $\hat{z}$ conditional on becoming a homeowner.

A mortgage term of 25 years is assumed ($T = 25$), and an average loan-to-value ratio of 80% ($\ell = 0.8$). Focusing on interest rates fixed for five years as a typical mortgage product, the 5-year conventional mortgage rate from Stats Canada was 7.07% in 2007. Given an inflation rate of 2.14%, the implied real mortgage rate is set to 4.93% for a median buyer. Information on mortgage spreads is then used to compute credit costs for the marginal buyer. Based on the micro-level mortgage data from the Bank of Canada, the average contract mortgage rate during 2017–2018 was around 3.11%. Borrowers with low credit scores and therefore unqualified for major banks could obtain mortgages from Trust Companies or private lenders at mortgage rates of around 6.15%. Hence, a mortgage spread of 3% is assumed, implying the real mortgage rate for the marginal buyer is 7.93%. Since median mortgage cost is based on 5-year fixed rates, the equivalent risk-free rate is derived from 5-year government bonds. These had a yield of 4% in 2007, so the real risk-free rate is set to 1.86%.

In summary, z/P is derived from the formula for χ/p using $T = 25$, $\ell = 0.8$, $r_f = 1.86\%$, $r_m = 4.93\%$, and the value of ρ obtained from the whole calibration method. The value of $z/\hat{z}$ is obtained by taking the ratio of χ/p for $r_m = 7.93\%$ and $r_m = 4.93\%$, with the other terms the same.

A.5 Calibration method

Transaction probabilities and selling and letting rates Using equations (59) and (60), the targets for V_o, V_l, T_{so}, and T_{sl} give the values of:

$$\pi_o = \frac{1}{V_o} \quad , \quad \pi_l = \frac{1}{V_l} \quad , \quad s_o = \frac{1}{T_{so}} \quad , \quad s_l = \frac{1}{T_{sl}}. \tag{A.58}$$

Allocation of housing stock Using equations (57), (60), and (61), it follows that $h_o/T_{mo} = u_o/T_{so}$, hence $u_o = (T_{so}/T_{mo})h_o$ and $h_o + u_o = (1 + T_{so}/T_{mo})h_o$. Similarly, $u_l = (T_{sl}/T_{ml})h_l$ and $h_l + u_l = (1 + T_{sl}/T_{ml})h_l$. Using the definition of the homeownership rate $h = h_o + u_o$ and (43) to note $h_l + u_l = 1 - h$, the targets for h

and times-on-the-market and times-to-move imply the housing stock is allocated according to:

$$h_o = \frac{h}{1+\frac{T_{so}}{T_{mo}}} \quad , \quad u_o = \frac{h\frac{T_{so}}{T_{mo}}}{1+\frac{T_{so}}{T_{mo}}} \quad , \quad h_l = \frac{1-h}{1+\frac{T_{sl}}{T_{ml}}} \quad , \quad u_l = \frac{(1-h)\frac{T_{sl}}{T_{ml}}}{1+\frac{T_{sl}}{T_{ml}}} . \tag{A.59}$$

Market tightness Using (44), (60), $s_o = \theta_o q_o \pi_o$, and the definition of market tightness $\theta_i = b_i/u_i$:

$$\theta_o = \frac{T_{bo}}{T_{so}} \quad , \quad \theta_l = (\phi - h_o - h_l - \theta_o u_o)/u_l \quad \text{and} \quad T_{bl} = \theta_l T_{sl} . \tag{A.60}$$

These use the targets for ϕ and T_{bo}, and the housing allocation derived in (A.59).The final equation gives the implied value of T_{bl}, which cannot be chosen freely given the other targets. The viewing rates for home-buyers and renters follow from (59) and (60):

$$q_o = \frac{V_o}{T_{bo}} \quad \text{and} \quad q_l = \frac{V_l}{T_{bl}} . \tag{A.61}$$

Transitions to homeownership Let ψ denote the target for the fraction of first-time buyers. Using the law of motion for home-buyers (52), the value of ψ in a steady state with $b_{oo} = b_o$ and $\dot{b}_{oo} = 0$ is:

$$\psi = \frac{(\rho\phi + a_l \varsigma h_l)G_m(z)}{n_o h_o + (\rho\phi + a_l \varsigma h_l)G_m(z)} = \frac{n_o h_o - (q_o \pi_o + \rho)b_o}{(q_o \pi_o + \rho)b_o} .$$

This can be calculated from the ratio of inflows because the two groups transact at the same rate conditional on entering the stock b_{oo}. The second expression for ψ follows because it is a steady state. Substituting $b_o = \theta_o u_o$, $s_o = \theta_o q_o \pi_o$, and using $s_o u_o = n_o h_o + \rho h_o$ from (57) shows that $\psi = \rho(h_o + \theta_o u_o)/((n_o+\rho)h_o + \rho\theta_o u_o)$. Dividing numerator and denominator by h_o, and using (61) and (A.60):

$$\psi = \frac{\rho\left(1+\frac{T_{bo}}{T_{mo}}\right)}{\frac{1}{T_{mo}} + \rho\frac{T_{bo}}{T_{mo}}} .$$

This can be rearranged to give the solution for ρ in terms of ψ and other known targets, and with this, the implied value of n_o can also be found from $n_o = (1/T_{mo}) - \rho$:

$$\rho = \frac{\psi}{T_{mo} + (1-\psi)T_{bo}} \quad \text{and} \quad n_o = \frac{(1-\psi)(T_{mo}+T_{bo})}{T_{mo}(T_{mo}+(1-\psi)T_{bo})} . \tag{A.62}$$

Using (61), observe that $n_l + \rho = T_{ml}^{-1}$. Taking the value of ρ from (A.62), $n_l = T_{ml}^{-1} - \rho$, and it can be checked whether this is positive. With (56), the parameter $a_l = n_l$ is obtained immediately.

Let a_{ho}, a_{hl}, a_{bo}, and a_{bl} be the average ages of the household heads of those in h_o, h_l, b_o, and b_l, and a_o and a_l the average ages of those in $h_o + b_o$ and $h_l + b_l$. The calibration target for the difference in the average ages of homeowners and renters is $\gamma = a_o - a_l$. Furthermore, let a_e and a_f denote the average age of new entrants to the city and first-time buyers respectively. Taking the group in $h_o + b_o$, exit occurs at rate ρ with first-time buyers of measure $\rho(h_o + b_o)$ arriving in steady state. The differential equation for the average age is thus $\dot{a}_o = 1 - \rho a_o + \rho a_f$. A steady-state age distribution therefore has $a_o = a_f + \rho^{-1}$. It is convenient to consider all ages relative to first entry to the city, which are denoted by $\alpha_o = a_o - a_e$, $\alpha_l = a_l - a_e$, and similarly for the other groups. The definition of γ and the average homeowner versus first-time buyer result imply:

$$\gamma = \alpha_o - \alpha_l \quad \text{and} \quad \alpha_o = \alpha_f + \rho^{-1} . \tag{A.63}$$

Now consider the group h_l. There is exit at rate $n_l + \rho$ and entry $q_l \pi_l b_l / h_l = (n_l + \rho)$ as a proportion of the group (see 58) from b_l where the average age is a_{bl}. Thus, $1 = (n_l + \rho)(a_{hl} - a_{bl})$ and hence:

$$\alpha_{hl} = \alpha_{bl} + (n_l + \rho)^{-1} . \tag{A.64}$$

Since $a_l = (h_l/(h_l+b_l))a_{hl} + (b_l/(h_l+b_l))a_{bl}$ by definition, it follows that $a_{hl} - a_l = (b_l/(h_l+b_l))(a_{hl} - a_{bl})$, and by using (61) and (A.64):

$$\alpha_{hl} = \alpha_l + \frac{b_l}{h_l + b_l} T_{ml} \,. \tag{A.65}$$

For the group b_l, there are outflows at rate $q_l\pi_l + \rho$, and inflows of proportion $(\rho\phi(1 - G_m(z)))/b_l$ from outside the city (average age a_e) and of proportion $a_l(1 - \varsigma G_m(z))h_l/b_l$ from h_l (average age a_{hl}), hence:

$$1 + \frac{\rho\phi(1 - G_m(z))}{b_l} a_e + \frac{a_l(1 - \varsigma G_m(z))h_l}{b_l} a_{hl} = (q_l\pi_l + \rho)a_{bl} \,.$$

Using $\rho\phi(1 - G_m(z)) = (q_l\pi_l + \rho)b_l - a_l(1 - \varsigma G_m(z))h_l$ from (63), this becomes $b_l = a_l(1 - \varsigma G_m(z))h_l\alpha_{hl} = (q_l\pi_l + \rho)b_l\alpha_{bl}$. Substituting (A.64) and using (63) again leads to $\rho\phi(1 - G_m(z))\alpha_{hl} = b_l + (q_l\pi_l + \rho)b_l T_{ml}$, noting (61). With $\theta_l = b_l/u_l$, $s_l = \theta_l q_l \pi_l$, and $s_l u_l = h_l/T_{ml}$ from (58) and (61), $(q_l\pi_l + \rho)b_l T_{ml} = (h_l/T_{ml})T_{ml} + \rho b_l T_{ml} = h_l + \rho b_l T_{ml}$, and by putting these equations together:

$$\alpha_{hl} = \frac{(h_l + b_l) + \rho b_l T_{ml}}{\rho\phi(1 - G_m(z))} \,. \tag{A.66}$$

Finally, consider the ages of first-time buyers. Using (52), a fraction $a_l\varsigma h_l G_m(z)/(\rho\phi + a_l\varsigma h_l)G_m(z)$ come from h_l, and a fraction $\rho\phi G_m(z)/(\rho\phi + a_l\varsigma h_l)G_m(z)$ are new entrants to the city. Therefore, $a_f = (a_l\varsigma h_l/(\rho\phi + a_l\varsigma h_l))a_{hl} + (\rho\phi/(\rho\phi + a_l\varsigma h_l))a_e$, which can be written as:

$$\alpha_f = \alpha_{hl} - \frac{\rho\phi}{\rho\phi + a_l\varsigma h_l}\alpha_{hl} = \alpha_{hl} - \frac{(h_l + b_l) + \rho b_l T_{ml}}{(\rho\phi + a_l\varsigma h_l)(1 - G_m(z))} \,, \tag{A.67}$$

where the second expression substitutes from (A.66). Using (58) and (63) again to write $(\rho\phi + a_l\varsigma h_l)(1 - G_m(z)) = (q_l\pi_l + \rho)b_l - a_l(1 - \varsigma)h_l = s_l u_l + \rho b_l - a_l(1 - \varsigma)h_l = (a_l + \rho)h_l + \rho b_l - a_l(1 - \varsigma)h_l = \rho(h_l + b_l) + n_l\varsigma h_l$. Substituting this and (A.65) into (A.67):

$$\alpha_f = \alpha_l + \frac{b_l T_{ml}}{h_l + b_l} - \frac{(h_l + b_l) + \rho b_l T_{ml}}{\rho(h_l + b_l) + n_l\varsigma h_l} = \alpha_l + \frac{T_{bl}T_{ml}}{T_{ml} + T_{bl}} - \frac{1 + \rho\frac{T_{bl}T_{ml}}{T_{ml} + T_{bl}}}{\rho + \varsigma n_l \frac{T_{ml}}{T_{ml} + T_{bl}}} \,, \tag{A.68}$$

where the second expression makes use of (A.59) and (A.60). Combining this formula with the two equations in (A.63) and simplifying yields the difference in average ages:

$$\gamma = \left(1 + \rho\frac{T_{ml}T_{bl}}{T_{ml} + T_{bl}}\right)\left(\frac{1}{\rho} - \frac{1}{\rho + n_l\frac{T_{ml}}{T_{ml} + T_{bl}}\varsigma}\right) \,.$$

Since ρ is known from earlier, this can be rearranged to give an equation for ς in terms of the targets:

$$\varsigma = \frac{\gamma\rho^2(T_{ml} + T_{bl})^2}{((1 - \gamma\rho)(T_{ml} + T_{bl}) + \rho T_{bl}T_{ml})n_l T_{ml}} \,. \tag{A.69}$$

Furthermore, the targets pin down the value of $G_m(z)$. Since $(\rho\phi + n_l\varsigma h_l)(1 - G_m(z)) = \rho(h_l + b_l) + n_l\varsigma h_l$ as shown above, the value of $G_m(z)$ must satisfy:

$$G_m(z) = \frac{\rho\phi - \rho(h_l + b_l)}{\rho\phi + n_l\varsigma h_l} \,, \tag{A.70}$$

and all the terms in this expression are known.

Discount rate and bargaining power Dividing both sides of the price equation (A.41) by P, and both sides of the rent equation (A.45) by R and rearranging:

$$\frac{\omega_o^*\Sigma_o}{\pi_o P} = \frac{(1 - c_u)r + m}{r + T_{so}^{-1}} \,, \quad \text{and} \quad \frac{\omega_l\Sigma_l}{\pi_l R} = \frac{1 - m_l - \omega_l(r + T_{ml}^{-1})(c_w + c_l)}{r + T_{ml}^{-1} + T_{sl}^{-1}} \,. \tag{A.71}$$

These objects are known given the calibration targets. The first equation is derived using (53) and (60) and the definitions $c_u = C_u/P$ and $m = M/P$. The second equation is derived using (53), (56), (60), and (61) together with the definitions $m_l = M_l/R$, $c_w = C_w/R$, and $c_l = C_l/R$. Dividing both sides of the indifference condition

(A.47) by P, it can be written in terms of the variables in (A.71) and the other calibration targets:

$$\frac{(1-\omega_o)}{\omega_o}\frac{(1+\tau_h)}{T_{bo}}\left(\frac{\omega_o^*\Sigma_o}{\pi_o P}\right)=\frac{1}{pT_{bl}}\left(\frac{1-\omega_l}{\omega_l}\right)\left(\frac{\omega_l\Sigma_l}{\pi_l R}\right)+(r+\rho)\frac{z}{P}+\left(1-\frac{F_w}{F_h}\right)f_h.$$

The derivation here uses $(1-\omega_o^*)/\omega_o^*=(1+\tau_h)(1-\omega_o)/\omega_o$ implied by (19), $q_o\pi_o=T_{bo}^{-1}$ and $q_l\pi_l=T_{bl}^{-1}$ from (60), and the definitions $p=P/R$ and $f_h=F_h/P$. Conditional on r and the other calibration targets including ω_l, this equation has a unique solution for the bargaining power ω_o:

$$\omega_o=\left(1+\frac{\frac{(1-m_l-\omega_l(r+T_{ml}^{-1})(c_w+c_l))}{pT_{bl}(r+T_{sl}^{-1}+T_{ml}^{-1})}+(r+\rho)\frac{z}{P}+\left(1-\frac{F_w}{F_h}\right)f_h}{\frac{(1+\tau_h)((1-c_u)r+m)}{T_{bo}(r+T_{so}^{-1})}}\right)^{-1},\tag{A.72}$$

where the expressions from (A.71) have been substituted here. Using ω_o from (A.72), the parameter ω_k is obtained using the relative bargaining power ω_k/ω_o calibration target.

The condition for free entry (A.46) can be expressed as follows, multiplying both sides r/P and noting that (18) implies $(1+\tau_k\omega_k^*)/(1-\omega_k^*)=1/(1-\omega_k)$:

$$\frac{\theta_l q_l\pi_l}{p}\left(\frac{\omega_l\Sigma_l}{\pi_l R}\right)-(1+\tau_k)\theta_o q_o\pi_o\left(\frac{\omega_o^*\Sigma_o}{\pi_o P}\right)=(c_k+(1+\tau_k)c_u)r-\tau_k m+\frac{f_k r}{(1-\omega_k)q_o},\tag{A.73}$$

which uses the definitions $p=P/R$, $c_k=C_k/P$, and $f_k=F_k/P$. Substituting from (A.71), this can be rearranged to give an expression for the price-rent ratio p:

$$p=\frac{\frac{1-m_l-\omega_l(r+T_{ml}^{-1})(c_w+c_l)}{T_{sl}(r+T_{sl}^{-1}+T_{ml}^{-1})}}{\frac{(1+\tau_k)((1-c_u)r+m)}{T_{so}(r+T_{so}^{-1})}+(c_k+(1+\tau_k)c_u)r-\tau_k m+\frac{f_k rT_{bo}}{(1-\omega_k)V_o}},\tag{A.74}$$

which is derived using (53), (59), and (60). This depends on known calibration targets and r, and also ω_k which can be determined using (A.72) and the target for ω_k/ω_o. As p is itself a target, this equation can be solved numerically to determine the discount rate r. Once r that solves (A.74) is found, the bargaining powers ω_o and ω_k are also determined (ω_l is a calibration target), as are ω_o^* and ω_k^* using (19) and (39).

Meeting functions With ω_o and ω_l known, the meeting function elasticities η_o and η_l are derived from the calibration targets for ω_o/η_o and ω_l/η_l. Since market tightnesses are already determined in (A.60) and the viewing rates in (A.61), the meeting function efficiency parameters A_o and A_l are those satisfying (12):

$$A_o=q_o\theta_o^{\eta_o}\quad\text{and}\quad A_l=q_l\theta_l^{\eta_k}.$$

Ownership market match quality and shocks All payoffs can be scaled without loss of generality so that the minimum new match quality in the ownership market is 1, hence $\zeta_o=1$. A new variable β_o is introduced at this stage, which is defined as follows in terms of the other parameters and endogenous variables of the model:

$$\beta_o=\frac{\lambda_o a_o\delta_o^{\lambda_o}\left(\frac{y_o}{x_o}\right)^{\lambda_o}}{\rho+a_o\left(1-\delta_o^{\lambda_o}\right)}.\tag{A.75}$$

It is supposed at this stage there is a target value of β_o alongside the other targets, but at the final stage, it is described how a target value of β_o can be derived from the econometric evidence on the response of transactions to the LTT change.

Consider the following numerical procedure to determine the arrival rate a_o of idiosyncratic shocks. Equation (62) implies the steady-state moving rate n_o can be written in terms of a_o, β_o, and λ_o:

$$n_o=\frac{a_o-\rho\frac{\beta_o}{\lambda_o}}{1+\frac{\beta_o}{\lambda_o}}.$$

The value of λ_o can then be determined conditional on the value of a_o and the provision target for β_o:

$$\lambda_o = \frac{(n_o + \rho)\beta_o}{a_o - n_o},$$

where the values of ρ and n_o come from (A.62). Next, take equation (A.39) and divide both sides by P:

$$\frac{x_o}{P} = \frac{((1-c_u)r + m)(1 - \omega_o^* + \omega_o^* \theta_o)}{\omega_o^*(r + T_{so}^{-1})T_{bo}} - f_h, \tag{A.76}$$

where (61) and (A.71) have been used. Similarly, dividing both sides of (A.38) by P:

$$\frac{y_o}{P} = \frac{x_o}{P} + (r + \rho + a_o)\left(c_h + (1+\tau_h)c_u - \tau_h \frac{m}{r} + \frac{(1-c_u)r + m}{r(1 + rT_{so})}\right), \tag{A.77}$$

which uses (53) and (60). Together, (A.76) and (A.77) imply that $y_o/x_o = (y_o/P)/(x_o/P)$ can be derived in terms of a_o and the calibration targets. With λ_o and y_o/x_o, equation (A.75) can be rearranged to solve for the idiosyncratic shock size parameter δ_o:

$$\delta_o = \left(\frac{\left(1 + \frac{\rho}{a_o}\right)\beta_o}{\beta_o + \lambda_o \left(\frac{y_o}{x_o}\right)^{\lambda_o}}\right)^{\frac{1}{\lambda_o}},$$

Using $\pi_o = (\zeta_o/y_o)^{\lambda_o}$ and (59), it follows that $y_o = \zeta_o V_o^{1/\lambda_o}$, so y_o is known given ζ_o, V_o, and λ_o. With the ratio y_o/P in (A.77), the price P is obtained, and hence the cost parameters $C_h = c_h P$, $C_u = c_u P$, $C_k = c_k P$, $F_h = f_h P$, $F_k = f_k P$, $M = mP$ and also x_o using (A.76). Since these parameters are all computed conditional on a conjectured value of a_o, the value of a_o is verified by a numerical search to check where the following equation holds:

$$x_o + F_h - \frac{(1 - \omega_o^* + \omega_o^* \theta_o) q_o \zeta_o^{\lambda_o}}{(1 + \tau_h \omega_o^*)(r + \rho + a_o)(\lambda_o - 1)}\left(y_o^{1-\lambda_o} + \frac{a_o \delta_o^{\lambda_o}}{r + \rho + a_o(1 - \delta_o^{\lambda_o})} x_o^{1-\lambda_o}\right) = 0. \tag{A.78}$$

This equation is derived from (A.39) and (A.40), and can be checked because everything in the equation is known conditional on a_o and the calibration targets. The condition $\delta_o y_o < x_o$ can also be verified at this stage.

Distribution of credit costs The value of $G_m(z)$ has already been determined in (A.70). Using (14), the marginal credit cost z and the parameters μ and σ satisfy:

$$\frac{\log z - \mu}{\sigma} = \Phi^{-1}(G(z)).$$

With the target for z/P and the known value of P, the value of the marginal credit cost z is determined. The conditional median credit cost $\hat{z}$ satisfies $\Phi((\log \hat{z} - \mu)/\sigma)/\Phi((\log z - \mu)/\sigma) = 1/2$, so together with (14):

$$\frac{\log \hat{z} - \mu}{\sigma} = \Phi^{-1}(G(z)/2).$$

Subtracting $\log \hat{z} = \mu + \sigma\Phi^{-1}(G(z)/2)$ from $\log z = \mu + \sigma\Phi^{-1}(G(z))$ and solving for σ and μ yields:

$$\sigma = \frac{\log(z/\hat{z})}{\Phi^{-1}(G(z)) - \Phi^{-1}(G(z)/2)} \quad \text{and} \quad \mu = \log z - \sigma\Phi^{-1}(G(z)).$$

Hence, μ and σ are known given a target for $z/\hat{z}$ in addition to z/P. The mean credit cost $\bar{z}$ can then be computed using (15).

Rental-market parameters Since P was determined earlier, the target for the price-to-rent ratio $p = P/R$ yields the average rent R. The targets for m_l, c_w, and c_l then imply values of the cost parameters $M_l = m_l R$, $C_w = c_w R$, and $C_l = c_l R$. The target for F_w/F_h can be used to obtain F_w from the value of F_h derived earlier.

With π_l known from (A.58), the value of Σ_l can be deduced from (A.71) using R and the calibration target ω_l. Equation (A.44) then implies $y_l = M_l - F_w + (r + a_l + \sigma_l + \rho)(C_w + C_l) - \sigma_l G(z)(z - \bar{z}) + (1 - \omega_l + \omega_l \theta_l) q_l \Sigma_l$. Since $\pi_l = (\zeta_l / y_l)^{\lambda_l}$, equation (A.30) can be rearranged to solve for λ_l in terms of y_l, π_l, and Σ_l:

$$\lambda_l = 1 + \frac{\pi_l y_l}{(r + \rho + \sigma_l + al)\Sigma_l} .$$

Finally, knowing λ_l allows the minimum new match quality parameter to be deduced from the equation for π_l as $\zeta_l = y_l \pi_l^{1/\lambda_l}$.

Matching the response of transactions to the LTT All the targets have been matched conditional on the value of β_o from (A.75), for which there was no explicit target. A numerical search over β_o values is then used to match the model's predicted response of transactions to the LTT with the econometric evidence on the transactions response.